KINGDOM OF SAUDI ARABIA

Ministry of Higher Education

Imam Mohammad Ibn Saud Islamic University

College of Science

Department of Mathematics & Statistics

1434/1435 Second Semester

Course Code &Number: Mat 321

Course Title: Modern Algebra

Date of Exam:: 22-6-1435H

Midterm 1

Duration: 1H15

Answer <u>FIVE</u> questions of the following :

1- Prove or disprove the following:

(a) A group of order 25 may have a subgroup of order 10.

(b) Any group of order 12 must have an element of order 6.

(c) $H = \{e, (1 \quad 4)\}$ is a normal subgroup of S_4.

(d) The index of the subgroup $\langle i \rangle$ of the quaternion group Q_8 is equal 2.

2- Find two subgroups of D_4 each of order 2 one is normal and one is not.

3- Find the distinct left cosets of the subgroup $H = \{e, \alpha, \alpha^2, \alpha^3\}$ of D_4.

4- Find the zero divisors of the ring $\mathbb{Z}_{20}$.

5- Let $S = \{0, 2, 4, 6, 8, 10, 12\}$ be a subring of $\mathbb{Z}_{14}$:

(a) Show that S has unity.

(b) Which elements of S have multiplicative inverses?.

6- Is $S = \left\{ \begin{bmatrix} x & x \\ 2x & 2x \end{bmatrix} : x \in \mathbb{Z} \right\}$ a subring of $M_2(\mathbb{Z})$?.

7- Prove that the order of an element of a finite group divide the order of the group.