MidTerm 2

Question 1 (2+3=5 marks)

Consider the vectors $u = (1, 1, 2)$, $v = (2, -2, 7)$ and $w = (3, 4, 5)$ in $\mathbb{R}^3$.

a) Determine the angle θ between u and v.

b) Determine whether u, v and w form a basis for $\mathbb{R}^3$.

Question 2 (1+2=3 marks)

a) Determine whether the following polynomials form a basis for the vector space $\mathbb{P}_2$ of polynomials of degree at most 2:

$$f(x) = 1, \ g(x) = x, \ h(x) = x^2, \ \text{and} \ k(x) = 1 + x + x^2.$$

b) Determine whether the set $U = \{(a, b) \in \mathbb{R}^2 \,|\, ab = 0\}$ is subspace of $\mathbb{R}^2$.

Question 3 (3+2=5 marks)

Let $U = \{(a, b, c) \,|\, a + 2b - c = 0\}$ be a subset of $\mathbb{R}^3$.

a) Show that U is a subspace of $\mathbb{R}^3$.

b) Find a basis for U.

Question 4 (2+3+2=7 marks)

Consider the following matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

a) Show that 8 is an eigenvalue of A.

b) Determine all the eigenvalues of the A.

c) Determine the eigenvectors corresponding to the eigenvalue 8.

GOOD LUCK