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EXAM DURATION 1H15

Midterm 2

10/12/2017

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**Question 1.** [2+2+2 marks]

Find the derivatives of the following functions:

$$1) f(x) = e^{\sin 4x}, \quad 2) g(x) = x^3(\cos^2 x + 2), \quad 3) h(x) = \frac{x^2}{\ln(x^2 + 2)}.$$

**Question 2.** [1+1 marks]

Consider the function  $f$  defined by  $f(x) = x^4 - x^3 + 1$ .

- 1) Verify that  $f$  satisfies the conditions of Rolle's theorem on the interval  $[0, 1]$ .
- 2) Find a value of  $c$  satisfying the conclusion of Rolles's theorem on the interval  $[0, 1]$ .

**Question 3.** [2 marks]

Use implicit differentiation to find  $y'$  if  $x^3y^3 - 2x^2 - 5y = 0$ .

**Question 4.** [1+2+1 marks]

Use l'Hôpital's rule to compute the following limits:

$$1) \lim_{x \rightarrow 2} \frac{\ln(3-x)}{x-2}, \quad 2) \lim_{x \rightarrow 0} \frac{\cos x - 3x^2 - 1}{x^2}, \quad 3) \lim_{x \rightarrow -\infty} \frac{3x}{e^{-2x}}.$$

**Question 5.** [1+1+2+2 marks]

Let  $f(x) = 3x^4 - 4x^3 + 2$ .

- 1) Find all critical numbers of the function  $f$ .
- 2) Find the absolute extrema of the function  $f$  on the interval  $[-1, 2]$ .
- 3) Determine the intervals on which the function  $f$  is increasing and the ones where  $f$  is decreasing.
- 4) Find the local extrema of the function  $f$ .