Kingdom of Saudi Arabia
Ministry of higher Education
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المملكة العربية السعودية
وزارة التعليم العالي
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم
قسم الرياضيات و الإحصاء

Department: Mathematics & Statistics
Semester/Year: First /1435-1436
Duration: 75 minutes

Course Elements of sets and structures
Course Code: MAT 220

Midterm 1

QUESTION 1 [8=3+3+2 marks]

Let P, Q and R be three statements.

1. Use the truth table to prove that the following compound statements are logically equivalent: $\neg(P \wedge \neg Q) \Rightarrow P \equiv P$
2. Prove the following logical equivalence using the stated laws (**without** truth table): $(P \Rightarrow Q) \Rightarrow Q \equiv P \vee Q$
3. Give the **inverse** of the conditional statement: $P \Rightarrow \neg(Q \vee R)$

QUESTION 2 [5=2+3 marks]

1- Determine whether the following statement is a tautology, a contradiction, or neither:

$(P \Rightarrow Q) \vee (Q \Rightarrow P)$

2. Let P(x) and Q(x) be open sentences in x with nonempty universe U.
Give the **negation** of quantified statement: $(\exists x)(P(x) \Rightarrow \neg Q(x))$

QUESTION 3 [7=3+2+2 marks]

1. Let m and n be integers. Prove that the integer $m^2(n^2 - 1)$ is even if and only if m is even or n is odd.

2. Use the proof by cases to show the statement: If n is an integer number, then $n^2 + 5n + 11$ is odd.
3. Prove, by the principle of mathematical induction, that:

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}, \quad \forall n \geq 1$$

Extra exercise (bonus) [3 marks]:
Prove, by the principle of mathematical induction, that:

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}, \quad \forall n \geq 1$$