EXAM DURATION 1H15 **Midterm 1** 05/11/2017

Question 1. [2+2+2+2 marks]

Evaluate the following limits:

1) $\lim\limits_{x\to -1} \dfrac{x^2+x}{x^2+3x+2}$ 2) $\lim\limits_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$ 3) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{9x^2+2}}{3x-6}$ 4) $\lim\limits_{x\to \infty} \dfrac{\sin x}{x^3}$.

Question 2. [3 marks]

Find all vertical and horizontal asymptotes of the function $f(x) = \dfrac{2x^2+1}{x^2+5x+4}$.

Question 3. [4 marks]

Consider the function f defined by

$$f(x) = \begin{cases} \dfrac{3(x-2)}{x^2-3x+2} & \text{if} \quad x<2 \\ 3 & \text{if} \quad x=2 \\ e^{x-2}+2 & \text{if} \quad x>2 \end{cases}$$

1) Evaluate $\lim\limits_{x\to 2^-} f(x)$, $\lim\limits_{x\to 2^+} f(x)$ and $\lim\limits_{x\to 2} f(x)$.

2) Study the continuity of the function f at the points $x=1$ and $x=2$.

Question 4. [1.5+2+1.5 marks]

1) Use definition of derivative to find the derivative f' of the function $f(x)=\sqrt{x-3}$.

2) Find the derivatives of the following functions:

$$a)\ f(x) = x\sqrt[3]{x} + \frac{1}{x^2} \qquad b)\ g(x) = \frac{\sqrt{x^4+3}}{x}.$$

3) Find the equation of tangent line to the graph of the function $f(x) = \dfrac{1}{x-1}$ at $a=2$.