

KINGDOM OF SAUDI
ARABIA

Ministry of Higher Education

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1434/1435 Second Semester

Course Code & Number: Mat 321

Course Title: Modern Algebra

Date of Exam:: 17-5-1435H

Midterm 1

Duration: 1H15

Answer **FIVE** questions of the following :

1- Find all subgroups and all generators of the additive group \mathbb{Z}_{30} .

2-(i) Prove that every cyclic group is Abelian.

(ii) Is the group $G = \{1, 3, 7, 9\}$ cyclic with respect to the operation \odot defined as $a \odot b = ab \pmod{10}$ for every $a, b \in G$.

3- (i) List all the elements of the subgroup $\langle [3] \rangle$ in the group \mathbb{Z}_{11}^* under multiplication, and state what is $O([4])$.

(ii) Prove or disprove: A finite group of even order contains an odd number of elements of order greater than 2.

4-(i) Show that $\varphi: \mathbb{R}^* \rightarrow \mathbb{R}^*$ defined by $\varphi(x) = \frac{|x|}{x}$ is an endomorphism and find its kernel and image.

(ii) If $\varphi: \mathbb{Z}_6 \rightarrow \mathbb{Z}_7^*$ is an isomorphism then what is the value of $\varphi([3])$ (i.e what is the image of the element $[3]$).

5- Consider the additive group \mathbb{Z}_{20} and define $f: \mathbb{Z}_{20} \rightarrow \mathbb{Z}_{20}$ by $f([x]) = [4x]$. Prove that f is a homomorphism and find $\ker f$. Is f an epimorphism? Is f a monomorphism?

6-(i) Express the following permutation as a product of disjoint cycles and then find its order and decide whether the permutation is even or odd. $\alpha = (1\ 3\ 2\ 4)(1\ 7\ 6\ 2\ 4)$.

(ii) Find $Z(S_3)$.