



Department: Mathematics & Statistics
Semester/Year: First /1436-1437

Course Elements of sets and structures
Course Code: MAT 220

Duration: 75 minutes

Midterm 1

NAME:

ID:

QUESTION 1 [8=3+2+3 marks]

Let P, Q and R be three statements.

1. Prove that the following compound statements are

logically equivalent: $(\neg P \wedge Q) \vee (P \wedge \neg Q) \equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$

2-Complete the following with *T* or *F* :

<i>P</i>	<i>Q</i>	<i>R</i>	$(P \vee Q) \Rightarrow \neg R$
<i>F</i>	<i>F</i>

3. Without using the truth table, prove that the following statement is tautology

$$[(P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow R)] \Rightarrow R$$

QUESTION 2 [5=2+2 marks]

1- Determine whether the following statement is a tautology, a contradiction, or neither:

$$[P \wedge (P \Rightarrow Q)] \Rightarrow Q$$

2. Let $P(x)$ and $Q(x)$ be open sentences in x with nonempty universe U .

Give the **negation** of quantified statement: $(\exists x)(P(x) \vee \neg Q(x))$

QUESTION 3 [7=3+2+3 marks]

1. Let m and n be integers. Prove that the integer $m^2 + n^2$ is even if and only if m and n are both even integers or m and n are both odd integers .

2. Let m and n be two integers. Prove, by a direct proof, that :
If m and n are both odd integers, then $5m + 7n + 2$ is an even integer..

3. Prove, by the principle of mathematical induction, that:

$$\frac{2}{1 \times 3} + \frac{2}{3 \times 5} + \cdots + \frac{2}{(2n-1) \times (2n+1)} = \frac{2n}{2n+1}, \quad \forall n \geq 1$$