Ministry of Education

Al-Imam Mohammed Ibn Saud Islamic University

College of Science

Department of Mathematics and Statistics

Midterm Exam

Course Name: Graph Theory and Combinatorics

Course Code: Mat 651

Semester/Year: Second/1437-1438

Date/Time: 07-08-1438 H/ 9:45 am

Duration: 2 Hours

Instructions: Ordinary calculators are allowed.

Answer the following questions:

Question 1 [**3+2+3=8 marks**]

(a) How many ways are there to arrange the letters in DIFFERENTITION with no pair of consecutive I's?

(b) How many bit strings of length 12 that contain at least three ones?

(c) Find a closed form for the generating function for the sequence $a_n = n^2 - 1$.

Question 2 [**3+4=7 marks**]

(a) How many ways are there to distribute six identical oranges and four distinct apples (each a different variety) into five distinct boxes? How many ways are there to distribute two objects in each box?

(b) Prove that the number of onto (surjective) functions from a set of m elements to a set of n elements is given by $\sum_{k=0}^{n-1} (-1)^k \binom{n}{k} (n-k)^m$.

Question 3 [**3+4=7 marks**]

(a) Find a recurrence relation for the number of ternary strings of length n that contain two consecutive 0s.

(b)Use the exponential generating functions to solve the recurrence relation

$$d_n = nd_{n-1} + 2^n, \quad n \geq 1, \text{ subject to } d_0 = 1..$$

Question 4 [**2+2+4=8 marks**]

(a) Use the identity $2\binom{n}{2} + 2\binom{n}{1} = n^2 + n$ to evaluate the sum $\sum_{k=1}^{n} k(k+1)$.

(b)How many nonnegative integer solutions are there to the equation $x_1 + x_2 + x_3 = 15$ such that $x_1 \leq 5$.

(c) Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2^n$ with initial conditions $a_0 = 4$ and $a_1 = 12$.

Extra question (bonus) [**3 marks**]

How many ways are there to pay a bill of 101 dollars using a currency with bills of values of 1 dollar, 2 dollars, and 5 dollars?

(*End Questions & Good Luck)*