Ministry of Education
Al-Imam Mohammed Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

Midterm 1

Course Name: Graph Theory and Combinatorics
Course Code: Mat 651
Semester/Year: Second/1436-1437
Date/Time: 22-6-1437 / 8:00 am
Duration: 120 min's

Instructions: Ordinary calculators are allowed.

Answer Two parts from each of the following questions:

Question 1 [4+4=8 marks]

(a) How many strings of five lowercase letters from the English alphabet contains the letters x and y?

(b) Build a generating function for the sequence $a_n = n^2$ and use this or any other way to find a closed form for the sum: $1^2 + 2^2 + \ldots + n^2$.

(c) Solve the recurrence relation $a_n - 3a_{n-1} - 4a_{n-2} = n$ subject to $a_0 = 1, a_1 = 2$.

Question 2 [3+3=6 marks]

(a) Find the coefficient of x^{20} in the expansion of $(x^2 + x^3 + x^4 + \ldots)^4$.

(b) Find the number of integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 40$ with $x_1 \geq 0$, $x_2 \geq 5$, $2 \leq x_3 \leq 7$ and $5 \leq x_4 \leq 10$.

(c) For $0 \leq k \leq m \leq n$, prove that $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$.

<u>Question 3</u> [4+4=8 marks]

(a)Given 6 different Arabic books, 5 different English books and 4 different French books. How many ways are there to make a row of three books in which exactly one language is missing (the order of the three books make a difference).

(b)Let $\Sigma = \{0,1,2,3,4\}$ be an alphabet. How many word of length 20 from Σ that contains an even number of zero's?

(c)How many positive integers not exceeding 5^6 that are square or cube?

<u>Question 4</u> [4+4=8 marks]

(a)Use the generating function to solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 2^{n-2}$ with initial conditions: $a_0 = 2$ and $a_1 = 8$.

(b)Find a closed form for the generating function for the following sequences:

(i) $a_n = \dfrac{1}{(n+1)!}$ (ii) $0, 1, -\dfrac{1}{2}, \dfrac{1}{3}, -\dfrac{1}{4}, \ldots$

(c) Evaluate the sum: $\sum_{k=0}^{n} \binom{r+k}{k}$ and if $0 \le m \le n$ then prove that $\sum_{k=0}^{n} \binom{n-k}{m-k} = \binom{n+1}{m}$.

<u>(End Questions & Good Luck)</u>