Ministry of Education
Al-Imam Mohammad Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

Course Name: Combin. and Graphs

Course Code: MAT 354

Semester/Year: First/1438-1439H

Date/Time: 14-04-1439H / 8:00 am

Duration: 2 Hours

Instructions: Only ordinary calculators are allowed.

Final Exam

1\01\2018

Name	ID	section

Q1		**8**
Q2		**8**
Q3		**6**
Q4		**10**
Q5		**8**
Total		**40**

Question 1:(**8=4+2+2 pts**)

a) How many strings of length four from the letters A, B, C, D, X, Y, and Z are there

i) if letters can be repeated?

ii) if no letter can be repeated?

iii) if each string contains the letter A?

iv) if each string contains the letters A and B?

b) Find the coefficient of x^{19} in the expansion of $(2x^3 - 3x)^9$.

c) A basket of fruits has 10 apples and 12 bananas. What is the smallest number of these fruits that should take out at random in the dark from the basket to guarantee that you have at least five pieces of the same fruit?

Question 2:(8=4+4 pts)

a) Solve the recurrence relation $a_n = 7a_{n-1} - 6a_{n-2}$ together with initial conditions $a_0 = 1$, $a_1 = 2$.

b) Let n and k be a positive integers with $k \leq n$. Prove that $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$.

Use this identity to simplify the following:

$$\binom{10}{5} + \binom{10}{6} + \binom{11}{7} + \binom{12}{8} + \cdots + \binom{19}{15} + \binom{20}{16}$$

Question 3: :(6=4+2 pts)

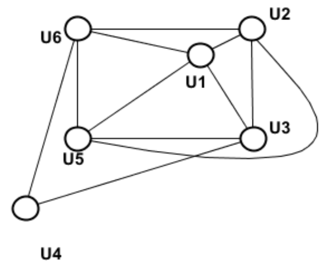
a) Use generating functions to solve the recurrence relation $a_n = 2a_{n-1} + 3.2^n$ together with the initial condition $a_0 = 3$.

b) There are 5 distinct white cars and 5 distinct black cars must be park in a line. How many ways are there to park these cars so that white and black cars alternate?

Question 4: (**10=2+2+2+2+2 pts**)

A graph G is given below:

1) Find the adjacency matrix of G



2) Find an Euler circuit in G.

3) Find a Hamilton circuit in G.

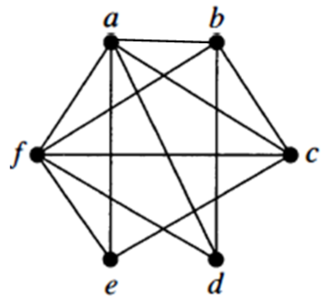
4) Find the chromatic number $\chi(G)$.

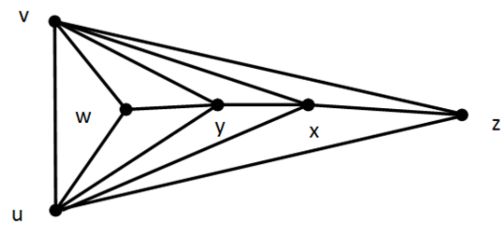
5) Is G planar? Justify your answer.

Question 5: (8=2+2+4 **pts**)

a) Can a simple graph exist with 3 vertices each of degree 3 and four vertices each of degree 5? Justify your answer.

b) Find an isomorphism between the graphs given below.





c) Use Dijkstra's algorithm to find a shortest part from the vertex **a** to the vertex **h**. What is length of this path?

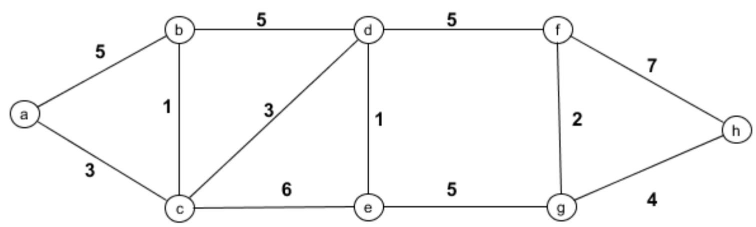


TABLE 1 Useful Generating Functions.	
$G(x)$	a_k
$(1+x)^n = \sum_{k=0}^{n} C(n,k)x^k = 1 + C(n,1)x + C(n,2)x^2 + \cdots + x^n$	$C(n,k)$
$(1+ax)^n = \sum_{k=0}^{n} C(n,k)a^k x^k = 1 + C(n,1)ax + C(n,2)a^2x^2 + \cdots + a^n x^n$	$C(n,k)a^k$
$(1+x^r)^n = \sum_{k=0}^{n} C(n,k)x^{rk} = 1 + C(n,1)x^r + C(n,2)x^{2r} + \cdots + x^{rn}$	$C(n,k/r)$ if $r \mid k$; 0 otherwise
$\frac{1-x^{n+1}}{1-x} = \sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$	1 if $k \le n$; 0 otherwise
$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots$	1
$\frac{1}{1-ax} = \sum_{k=0}^{\infty} a^k x^k = 1 + ax + a^2x^2 + \cdots$	a^k
$\frac{1}{1-x^r} = \sum_{k=0}^{\infty} x^{rk} = 1 + x^r + x^{2r} + \cdots$	1 if $r \mid k$; 0 otherwise
$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1)x^k = 1 + 2x + 3x^2 + \cdots$	$k+1$
$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)x^k = 1 + C(n,1)x + C(n+1,2)x^2 + \cdots$	$C(n+k-1,k) = C(n+k-1,n-1)$
$\frac{1}{(1+x)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)(-1)^k x^k = 1 - C(n,1)x + C(n+1,2)x^2 - \cdots$	$(-1)^k C(n+k-1,k) = (-1)^k C(n+k-1,n-1)$
$\frac{1}{(1-ax)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)a^k x^k = 1 + C(n,1)ax + C(n+1,2)a^2x^2 + \cdots$	$C(n+k-1,k)a^k = C(n+k-1,n-1)a^k$
$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$1/k!$
$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k} x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$(-1)^{k+1}/k$