Ministry of Education
Al-Imam Mohammad Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

Course Name: Combin. and Graphs
Course Code: MAT 354
Semester/Year: Second/1437-1438H
Date/Time: 26-08-1438H / 8:00 am
Duration: 2 Hours

Instructions: Only ordinary calculators are allowed.

Final Exam
22\05\2017

Name	ID	section

Q1		**8**
Q2		**8**
Q3		**8**
Q4		**8**
Q5		**8**
Total		**40**

Question 1:

a) (3pts) How many positive integers from 1000 and 5000 are not divisible by both 9 and 11**?**

b) (2pts) What is the minimum number of students in a school to be sure that at least six were born in the same month?

c) (3pts) What is the coefficient of x^{20} in the expansion of $(x^2 + \frac{2}{x^2})^{80}$?

Question 2 :

a) **(4pts)**Let n be a positive integer, prove the identity

$$\binom{2n}{n+1} + \binom{2n}{n} = \frac{1}{2}\binom{2n+2}{n+1},$$

b) (4pts) Suppose that a department contains 16 men and 12 women. How many ways are there to form a committee with eight members if it must have more men than women?

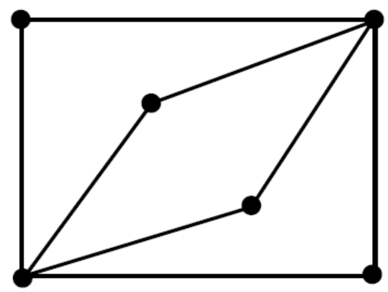
Question 3:

a) (4pts) Use generating functions to solve the recurrence relation $a_n = 3a_{n-1} + 2^{n-1}$ with $a_0 = 1$.

b) (4pts) Solve the homogeneous recurrence relation: $a_n = 7a_{n-1} - 12a_{n-2}$ together with initial conditions $a_0 = 0,\ a_1 = 2.$

Question 4:

Given the following graph G



a) (2pts) Find the adjacency matrix for G.

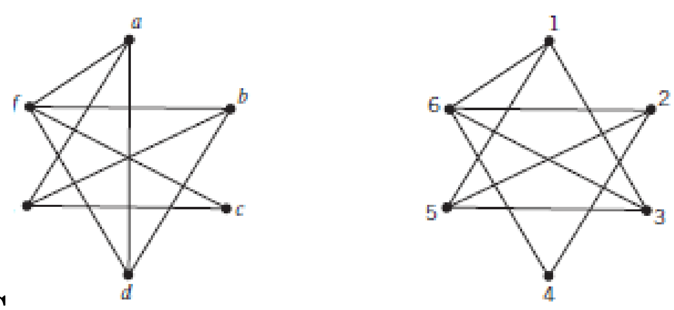
b) (2pts) What is $\chi(G)$?

c) (2pts) Has G Euler circuit? Justify your answer.

d) (2pts) Determine whether G has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

Question 5:

a) (2pts) Determine whether the graphs G and H displayed in the following figure are isomorphic



G **H**

b) (2pts) For which values of m and n the complete bipartite graph $K_{m,n}$ is Eulerian?

c) **(4pts)** Find the shortest path and its length between **a** and **z** in the given weighted graph.

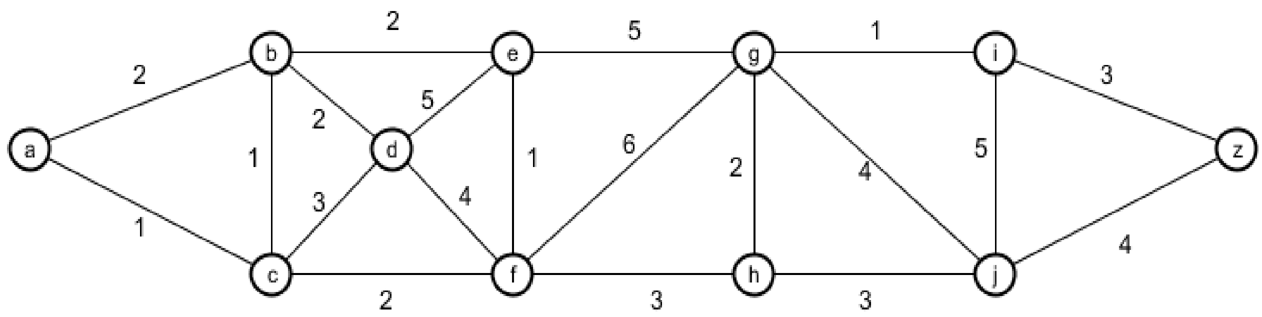


TABLE 1 Useful Generating Functions.

$G(x)$	a_k
$(1+x)^n = \sum_{k=0}^{n} C(n,k)x^k$ $= 1 + C(n,1)x + C(n,2)x^2 + \cdots + x^n$	$C(n,k)$
$(1+ax)^n = \sum_{k=0}^{n} C(n,k)a^k x^k$ $= 1 + C(n,1)ax + C(n,2)a^2x^2 + \cdots + a^n x^n$	$C(n,k)a^k$
$(1+x^r)^n = \sum_{k=0}^{n} C(n,k)x^{rk}$ $= 1 + C(n,1)x^r + C(n,2)x^{2r} + \cdots + x^{rn}$	$C(n, k/r)$ if $r \mid k$; 0 otherwise
$\frac{1-x^{n+1}}{1-x} = \sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$	1 if $k \le n$; 0 otherwise
$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots$	1
$\frac{1}{1-ax} = \sum_{k=0}^{\infty} a^k x^k = 1 + ax + a^2x^2 + \cdots$	a^k
$\frac{1}{1-x^r} = \sum_{k=0}^{\infty} x^{rk} = 1 + x^r + x^{2r} + \cdots$	1 if $r \mid k$; 0 otherwise
$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1)x^k = 1 + 2x + 3x^2 + \cdots$	$k+1$
$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)x^k$ $= 1 + C(n,1)x + C(n+1,2)x^2 + \cdots$	$C(n+k-1,k) = C(n+k-1,n-1)$
$\frac{1}{(1+x)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)(-1)^k x^k$ $= 1 - C(n,1)x + C(n+1,2)x^2 - \cdots$	$(-1)^k C(n+k-1,k) = (-1)^k C(n+k-1,n-1)$
$\frac{1}{(1-ax)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)a^k x^k$ $= 1 + C(n,1)ax + C(n+1,2)a^2x^2 + \cdots$	$C(n+k-1,k)a^k = C(n+k-1,n-1)a^k$
$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$1/k!$
$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k} x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$(-1)^{k+1}/k$