Ministry of Education
Al-Imam Mohammad Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

Course Name: Combin. and Graphs
Course Code: MAT 354
Semester/Year: First/1437-1438H
Date/Time: 18-04-1438H / 8:00 am
Duration: 2 Hours

Instructions: Only ordinary calculators are allowed.

Final Exam
16\01\2017

Name	ID	section

Q1		**8**
Q2		**6**
Q3		**8**
Q4		**6**
Q5		**12**
Total		**40**

Question 1:

a) **(3pts)** A test contains 100 true/false questions. How many ways a student can answer the questions on the test, if the answer may be left blank?

b) **(2pts)** How many people are needed to guarantee that at least two were born on the same day of the week?

c) **(3pts)** How many six characters password can be made from the 10 characters 1, 2, 3, 4, A, B, C, D, E,F, if the password contains at least one digit?

Question 2 :

a) **(4pts)** Prove the identity

$\binom{\boldsymbol{n}}{\boldsymbol{r}}\binom{\boldsymbol{r}}{\boldsymbol{k}} = \binom{\boldsymbol{n}}{\boldsymbol{k}}\binom{\boldsymbol{n-k}}{\boldsymbol{r-k}}$ where n, k and r are nonnegative integers with $r \le n \ and \ k \le r$.

b) **(2pts)** A man has 10 identical toys to distribute among three distinct children. How many ways he distributes the toys such that each child receives at least 3 toys and not more than 5 toys?

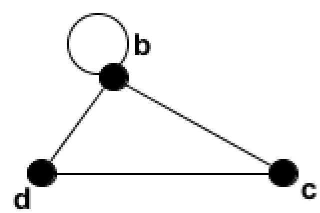
Question 3:

a) **(4pts)** Use generating functions to solve the recurrence relation $a_n = ٢a_{n-١} + ٣a_{n-٢}$ with $a_٠ = ٧, a_١ = ١$.

b) **(4pts)** Solve nonhomogeneous recurrence relation $a_n = ١٠a_{n-١} - ٢٥a_{n-٢} + ٢^n$ together with the initial conditions $a_٠ = ٣$ and $a_١ = ١٧$.

Question 4:

Given the following graph G



a) (2pts) Give the adjacency matrix for G.

b) (2pts) How many paths from b to d are of length 3?

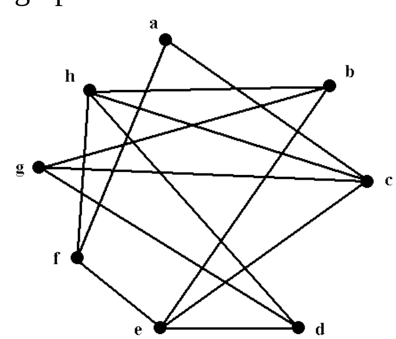
c) (2pts) Is G Eulerian? Justify your answer.

Question 5:

a) **(2pts)** A connected planar simple graph with 150 vertices and 200 edges divides the plane into how many regions?

b) **(2pts)** For which value of n the complete graph K_n is Eulerian?

c) **(4pts)** Is the graph drawn below planar? If it is planar‹ redraw it without edges crossing, if no explain why? What is the chromatic number of this graph?



d) **(4pts)** find the shortest path and its length between **a** and **h** in the given weighted graph.

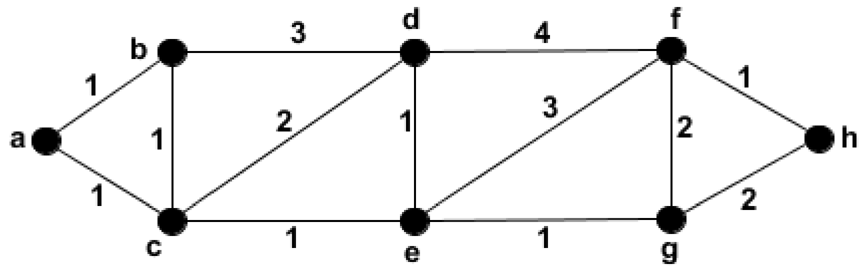


TABLE 1 Useful Generating Functions.	
$G(x)$	a_k
$(1+x)^n = \sum_{k=0}^{n} C(n,k)x^k = 1 + C(n,1)x + C(n,2)x^2 + \cdots + x^n$	$C(n,k)$
$(1+ax)^n = \sum_{k=0}^{n} C(n,k)a^k x^k = 1 + C(n,1)ax + C(n,2)a^2x^2 + \cdots + a^n x^n$	$C(n,k)a^k$
$(1+x^r)^n = \sum_{k=0}^{n} C(n,k)x^{rk} = 1 + C(n,1)x^r + C(n,2)x^{2r} + \cdots + x^{rn}$	$C(n,k/r)$ if $r \mid k$; 0 otherwise
$\frac{1-x^{n+1}}{1-x} = \sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$	1 if $k \le n$; 0 otherwise
$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots$	1
$\frac{1}{1-ax} = \sum_{k=0}^{\infty} a^k x^k = 1 + ax + a^2x^2 + \cdots$	a^k
$\frac{1}{1-x^r} = \sum_{k=0}^{\infty} x^{rk} = 1 + x^r + x^{2r} + \cdots$	1 if $r \mid k$; 0 otherwise
$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1)x^k = 1 + 2x + 3x^2 + \cdots$	$k+1$
$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)x^k = 1 + C(n,1)x + C(n+1,2)x^2 + \cdots$	$C(n+k-1,k) = C(n+k-1,n-1)$
$\frac{1}{(1+x)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)(-1)^k x^k = 1 - C(n,1)x + C(n+1,2)x^2 - \cdots$	$(-1)^k C(n+k-1,k) = (-1)^k C(n+k-1,n-1)$
$\frac{1}{(1-ax)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)a^k x^k = 1 + C(n,1)ax + C(n+1,2)a^2x^2 + \cdots$	$C(n+k-1,k)a^k = C(n+k-1,n-1)a^k$
$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$1/k!$
$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k} x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$(-1)^{k+1}/k$