Ministry of Education
Al-Imam Mohammed Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

Final Examination

Course Name: Appl. Calculus 1
Course Code: MAT 113
Semester/Year: First/1437-1438 H
Date/Time: 17-04-1438 H / 8:00 am
Duration: 2 Hours

Instructions: Only ordinary calculators are allowed.

Answer the following questions:

Question 1. [**10= (2+3+2)+3 marks**]: 1-Evaluate each of the following limits. All work must be shown.

$$(a) \quad \lim_{x\to 2^+} \frac{|x-2|}{x^2-x-2}$$

$$(b) \quad \lim_{x\to 0} \frac{\sin(3x)-3x-x^2}{1-\cos(2x)}$$

$$(c) \quad \lim_{x\to \infty} \frac{x^2+x}{xe^x+x}$$

2-Find the value of the constant c that makes the following function continuous

$$f(x)=\begin{cases} 2x+\dfrac{9}{x}, & if\ x\geq 3 \\ -4x+c, & if\ x<3 \end{cases}$$

Question 2. [**10= (3+2)+3+2 marks**]

1- Compute the first derivative for the following:

$$(a) \quad y=\ln(x^3+9)+\tan^{-1}(x^2).$$

$$(b) \quad y=\sqrt{(x^2+1)(x^4+1)}.$$

2- Find $y'(x)$ for $x^2-xy+y^2=7$. Then, find an equation of the tangent line at the point (-1,2).

3- Find a value of c satisfying the conclusion of the Rolle's Theorem for $f(x)=x^3-x+1$ on $[0, 1]$.

Question 3.[10 marks] Consider the function $f(x) = x^4 - 6x^2 + 8x + 12$. Its derivative is $f'(x) = 4(x+2)(x-1)^2$.

(a) Find the critical numbers of $f(x)$. Determine the absolute extrema of $f(x)$ on the interval [0,2].

(b)Determine the intervals where the given function is increasing and decreasing.

(c) Determine all local extrema of the given function.

(d)Find the intervals where the graph of given function is concave up and concave down.

(e)Find the infection points of $f(x)$.

Question 4. [10=8+2 marks]

1- Evaluate each of the following integrals, showing all reasoning.

(a) $\int_1^4 \frac{2x^2 + x + 4}{x}\,dx$ (b) $\int \frac{x^3}{x^4 + 1}\,dx$

(c) $\int_1^{} x^2 \cos(x^3 + 1)dx$ (*Use a substitution with* $u = x^3 + 1$)

(d) $\int 2x\sqrt{x^2 - 9}\ dx.$

2- If $f(x) = \int_5^{x^2} \sqrt{1 + t^2}\ dt$, find $f'(x)$.

End of Exam.