Ministry of Education
Al-Imam Mohammed Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

Final Examination

Course Name: Calculus I
Course Code: MAT 101
Semester/Year: Second/1436-1437
Date: 01-08-1437
Duration: 120 min's

Instructions: Only ordinary calculators are allowed.

Answer the following questions

Question 1 [8=4x2 marks]: Evaluate each of the following limits. All work must be shown.

(a) $\quad \lim\limits_{x \to 2^+} \dfrac{\left|x-2\right|}{x^2 - 7x + 10}$
$\qquad (b)$ $\quad \lim\limits_{x \to \infty} \dfrac{x^3 + 8x}{x^2 + e^x}$

(c) $\quad \lim\limits_{x \to 0} \dfrac{x^2}{e^x - x - 1}$
$\qquad (d)$ $\quad \lim\limits_{x \to 0} \dfrac{1 - \cos(4x)}{x^2}$.

Question 2 [14=4x2+2+4 marks]:

(1) Find the first derivatives of the following functions:

(a) $\quad f(x) = x^{-3} + 3^x$
$\qquad (b)$ $\quad f(x) = \sin^{-1}(x^2) + \tan(x)$

(c) $\quad f(x) = \dfrac{x^2 \cos(x)}{(x^2 + 9)}$
$\qquad (d)$ $\quad f(x) = \ln(x^2 + 4) + 8e^{-x^2}$.

(2) Find y', if $y = (\sin x)^x$.

(3) Find the slope of the tangent line to the curve below at the point $(1,2)$

$x^4 y^2 + 6x^5 - y^3 + 2x = 4$.

Question 3 [10=2+4x2 marks]:

(1) Find the absolute maximum and absolute minimum of $\quad f(x) = \dfrac{x}{x^2 + 1}$ on the interval $[0,3]$.

(2) Given the function : $f(x) = x^3 - 6x^2 + 9x + 1$

    (a) Find all critical numbers.

    (b) Find the intervals where the function is increasing or decreasing.

    (c) Find the local extrema.

    (d) Find the intervals where the graph of given function is concave up or concave down.

Question 4 [8=4x2 marks]:

(1) Evaluate each of the following integrals, showing all reasoning.

$$(a) \quad \int_2^3 \frac{2x^2 + 3x + 1}{x} \, dx \qquad\qquad (b) \quad \int \frac{x^3}{x^4 + 1} \, dx$$

$$(c) \quad \int 3x^2 \sin(x^3 + 1) dx \qquad\qquad (d) \quad \int x\sqrt{x^2 + 1} \, dx$$

Extra question (Bonus)[4 marks]: Given the function $f(x) = \dfrac{(x-1)^2}{(x+2)(x-4)}$ and its

first derivative $f'(x) = \dfrac{18(1-x)}{(x+2)^2(x-4)^2}$:

(a) Find all horizontal and vertical asymptotes, if any.

(b) Determine on what intervals $f(x)$ increasing or decreasing.

<p align="center">(End Questions & Good Luck)</p>