Ministry of Education
Al-Imam Mohammad Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

FINAL Examination

Course Name: Graph Theory and Combinatorics
Course Code: MAT 651
Semester/Year: Second/1437-1438
Date/Time: 18-08-1438 H/ 9 am
Duration: 3 Hours

Instructions: Ordinary calculators are allowed.

Answer the following questions:

Question 1 [3+4+2=9 marks]

(a) A number is defined as *non-descending* if each of its digits are in non-decreasing numerical order. For example, 22256 and 17789 are non-descending numbers, but 13654 and 96872 are not. How many five digit numbers are non-descending?

(b) How many permutations are there of the letters of COMBINATORICS? How many of these permutations begin and end with the letter O? How many of these permutations do NOT have two vowels adjacent to one another? How many of these permutations are there in which the five vowels are consecutive?

(c) For which values of m and n the complete bipartite graph $K_{m,n}$ is Hamiltonian?

Question 2 [4+4=8 marks]

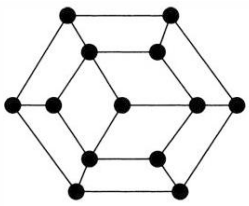
(a) Use generating functions to solve the following recurrence relation $a_n = 2a_{n-1} + 3^{n-1} + n$ with initial condition $a_0 = 0$.

(b) In how many ways can the vertices of a regular octagon (an 8 sided polygon) be colored using 2 colors in such a way that, two colorings are considered the same if one can be obtained from the other through rotation(s) and/or reflection(s).

Question 3 **[2+(1+2+1)+4=10 marks]**

(a) Evaluate the sum $\binom{n}{0}+2\binom{n}{1}+3\binom{n}{2}+4\binom{n}{3}+\cdots+(n+1)\binom{n}{n}$

(b)Consider the following graph G



(i) Find $\chi(G)$, the chromatic number of G.

(ii) Is G Hamiltonian? Justify your answer.

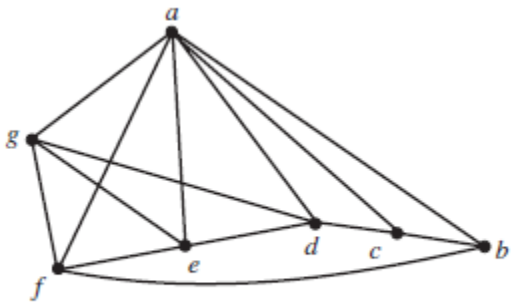
(iii) Is $\overline{G}$ planar? Justify your answer.

(c) Find the chromatic polynomial of the graph $K_{3,3}+e$ ($K_{3,3}$ plus an edge e).

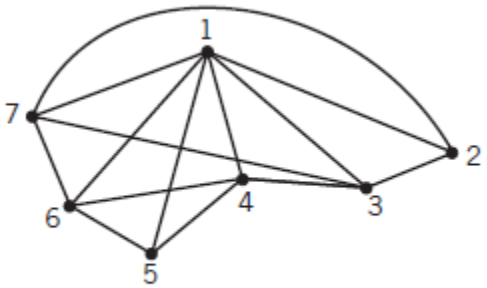
Question 4 **[2+2+3=7 marks]**

(a)Find the adjacency spectrum of the complete graph K_5 .

(b) Determine whether the following two graphs are isomorphic



G



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(c) How many numbers between 0 and 10,000 have a sum of digits equal to 13?

Question 5 [**6x1=6 marks**]

Prove or disprove the following:

(a) If G is a connected simple Eulerian graph of odd order, then $\bar{G}$ is Eulerian graph.

(b) If G is a simple connected graph of order n whose the degree of every vertex is greater than or equal $\frac{n}{2}$, then G is Hamiltonian.

(c) The chromatic polynomial of the cycle C_n is $(k-1)^n + (-1)^n(k-1)$.

(d) If G is a any simple connected graph with 6 vertices, then either G or $\bar{G}$ is non planar.

(e) Number of positive integers not exceeding 1000,000 that are square or cube is equal to 1100.

(f) Every Eulerian bipartite graph has an even number of edges.

Extra question (bonus) [**4 marks**]

Find the number of nonnegative integer solutions of the equation $x + y + z = 100$ such that $x = 0,1,2,3,\ldots$, $y = 0,5,10,15,\ldots$ and $z = 0,5,10,15,\ldots$.

(End Questions & Good Luck)