Ministry of Education
Al-Imam Mohammed Ibn Saud Islamic University
College of Science
Department of Mathematics and Statistics

Final Examination

Course Name: Graph Theory and Combinatorics
Course Code: Mat 651
Semester/Year: Second/1436-1437
Date/Time: 01-08-1437 H/ 9:00 am
Duration: 3 Hours

Instructions: Ordinary calculators are allowed.

Answer Two parts from each of the following questions:

Question 1 [**4+4=8 marks**]

(a) How many ways are there to distribute five different books among four children if every child take at least one book?

(b) How many bit string of length 10 that contains:

(i) at least three ones?

(ii) at most three ones?

(c) Use the generating functions to solve the recurrence relation

$a_n = 5a_{n-1} - 4a_{n-2}, \quad n \geq 2,$ subject to $a_0 = 1, a_1 = 2$.

Question 2 [**4+4=8 marks**]

(a) Use the exponential generating function to solve the recurrence relation $a_n = na_{n-1} + (-1)^n, \quad n \geq 1$ with initial condition $a_0 = 1$.

(b) How many different 10-bead necklaces are there using beads of red, white, and blue incase of rotations being considered equal.

(c) Let G be a simple connected graph with at least 11 vertices. Prove that either G or $\overline{G}$ is non planar.

Question 3 [**4+4=8 marks**]

(a)Show that $\sum_{k=m}^{n}\binom{k}{r}=\binom{n+1}{r+1}-\binom{m}{r+1}$.

(b)Find the chromatic polynomial for the graph $K_n - e$ (The graph obtained from the graph K_n by removing an edge).

(c)Show that a simple graph G of order n is connected if $\deg(v)\geq\frac{(n-1)}{2}$ for every v of G. Is this true in case $\deg(v)\geq\frac{(n-2)}{2}$ for every v of G?

Question 4 [**4+4=8 marks**]

(a)Show that every simple planar graph has a vertex of degree at most five and then prove that every simple planar graph is 6-colorable.

(b)Find the adjacency spectrum and the Laplacian spectrum of the complete graph K_4.

(c) Prove that every cubic Hamiltonian graph is 3-edge-colorable.

Question 5 [**4+4=8 marks**]

(a) For the generating function: $\frac{1+x^3}{(1+x)^3}$, provide a closed formula for the sequence it determines.

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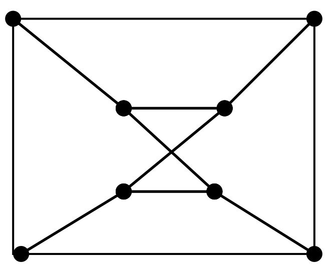
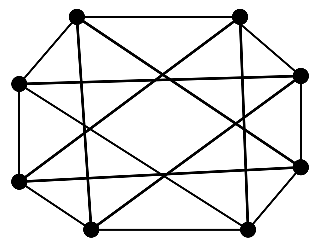
Let G and H be the following two graphs:



G



H

Justify your answer for the following:

(a) (i) Is G isomorphic to $\bar{H}$?

(ii) Is G Hamitonian?

(b) (i) Find $\chi(H)$ and $\chi_e(G)$.

(ii) Is $\bar{G}$ Eulerian?

(iii) Is H planar?

(End Questions & Good Luck)