



## Final Examination

### Question 1 . (6 Marks)

Solve the following system of linear equations using Gauss-Jordan elimination :

$$\begin{cases} x_1 - 2x_2 + 5x_3 - 5x_4 = -7 \\ 3x_1 + x_2 + x_3 + 6x_4 = 14 \\ 4x_1 + x_2 + 2x_3 + 7x_4 = 17 \end{cases}$$

### Question 2 . (6 Marks)

Find the inverse of the matrix :  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 7 \\ -1 & 1 & -4 \end{bmatrix}$ .

### Question 3 . (4+2=6 marks)

- (3.a) Determine whether the vectors  $(1, -2, 5)$ ,  $(3, 1, 1)$  and  $(4, 1, 2)$  form a basis for  $\mathbb{R}^3$ .
- (3.b) Determine whether the subset  $W = \{(2a, 2 + a) \mid a \in \mathbb{R}\}$  of  $\mathbb{R}^2$  is a subspace of  $\mathbb{R}^2$ .

### Question 4 . (3+3=6 marks)

Consider the matrix :  $B = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 0 \\ 5 & 5 & 5 \end{bmatrix}$ .

- (4.a) Determine all eigenvalues of  $B$ .
- (4.b) Determine all eigenvectors  $B$  corresponding to the eigenvalue  $\lambda = -1$ .

**Question 5** (4 marks for each part)

Solve the the following differential equations :

(5.a)  $\frac{dy}{dx} = \frac{1}{y}$  with  $y(0) = 1$ . (initial value).

(5.b)  $y' + \frac{1}{x}y = 1$ .

(5.c)  $y'' - 5y' + 6y = 0$ .

(5.d)  $y'' - 5y' + 6y = 6x + 1$ . (Hint : Use (5.c)).

**GOOD LUCK**