Kingdom of Saudi Arabia
Ministry of Education
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- College of Science -

المَملَكَة العَرَبِية السَعُودِية
وزَارَة التَّعليم
جَامِعَة الإِمَام مُحَمَّد بن سَعُود الإِسلَامِية
-كلّية العُلوم-

Semester: 143820

Linear Algebra **FINAL EXAM** MATH 226

Duration: 2H 30 min SECTION: Student Name ____________________

Question:	1	2	3	4	5	6	Total
Points:	8	7	8	8	4	5	40
Score:							

Answers written outside the allocated space will NOT be graded!!!
Calculators are not allowed

1. (a) [2 points] Let A be an $n \times n$ matrix satisfying $A^3 = 2A^2 + A$. Determine λ and δ such that $A^5 = \lambda A^2 + \delta A$.

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(b) [2 points] Show that if A is an antisymmetric 3×3 matrix , then A is not invertible.

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(c) [2 points] Prove that if a matrix A is an invertible $n \times n$ matrix, then $\text{adj}(A)$ is invertible. Determine $[\text{adj}(A)]^{-1}$.

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(d) [2 points] Let A be an $n \times n$ matrix. Prove that if 0 is an eigenvalue of A, then A is not invertible.

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2. 7 points Solve the following system regarding the values of λ :

$$\begin{cases} 2x_1 + (\lambda+3)x_2 + 11x_3 = -1 \\ x_1 - 2x_2 + 3x_3 = 0 \\ -x_1 + 3x_2 + \lambda x_3 = 1 \end{cases}.$$

3. (a) 4 points Determine the inverse (if it exists) of the matrix $A = \begin{bmatrix} -4 & 0 & 12 \\ -3 & 3 & 5 \\ -1 & -2 & 2 \end{bmatrix}$.

(b) 4 points Assume that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$. Evaluate the determinant $\begin{vmatrix} a & g & d \\ b & h & e \\ c+2a & i+2g & f+2d \end{vmatrix}$.

4. (a) 3 points Determine whether the subset $W = \{(x, y, z) \mid 2(x-1)+3(y+1)-(z+1) = 0\}$ is a subspace of $\mathbb{R}^3$.

(b) 4 points Determine whether the following set spans $\mathbb{R}^4$:

$$\{(1,-2,-4,3),(2,5,-2,9),(1,7,2,6),(0,5,-4,3),(2,-4,-8,6)\}.$$

(c) 1 point Use (b) to determine whether $W = \mathbb{R}^4$.

5. The only eigenvalues of the matrix $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ are 4 and 1.

(a) 3 points Prove that B is diagonalizable.

(b) 1 point Find an invertible matrix C such that $C^{-1}BC$ is diagonal.

6. (a) 2 points Determine the standard matrix of the linear transformation: $T_1 : \mathbb{R}^4 \to \mathbb{R}^3$ defined by $T_1((x, y, z, w)) = (x - y - z, 2y + 4z, x - y + z + w)$.

(b) Consider the linear transformation $T_2 : \mathbb{P}_3 \to \mathbb{P}_3$ defined by

$$T_2(ax^3 + bx^2 + cx + d) = (a + 2d)x^3 + (b + 2c)x^2 + (a + c + d)x.$$

i. 2 points Determine $\mathrm{Ker}(T_2)$.

ii. 1 point Determine a basis for range(T_2).