Kingdom of Saudi Arabia
Ministry of higher Education
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--- College of Science ---

بسم الله الرحمن الرحيم

المملكة العربية السعودية
وزارة التعليم العالي
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم
قسم الرياضيات و الإحصاء

Department: **Mathematics & Statistics**
Semester/Year: **First /1435-1436H**

Course: **Elements of sets and structures**
Course Code: MAT 220

Duration: 2 Hours

Final Examination

<u>QUESTION 1</u> [10=4+3+3 marks]

Let P, Q and R be three statements.

1- Prove the following logical equivalence: $(\neg P \Rightarrow (Q \Rightarrow R)) \equiv (Q \Rightarrow (P \vee R))$.

2- Show that the following statement is a tautology (**without** using the truth table):

$$((P \vee Q) \wedge \neg P) \Rightarrow Q.$$

3- Let $A = \{1,3,4,8\}$, $B = \{2,6,9\}$ and $C = \{1,2,4,5\}$ be subsets of the universal set $U = \{1,2,3,\ldots,10\}$. Determine: (a) $(A \cap C)'$ (b) $(A \cup B)'$ (c) $A - C$.

<u>QUESTION 2</u> [9=3+3+3 marks]

1- Prove, by the principle of mathematical induction, that:

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{n\times(n+1)}=\frac{n}{n+1}, \quad \forall n \geq 1.$$

2- Let m be an integer. Prove that $7m-4$ is odd if and only if $5m+3$ is an even integer.

3- Let A, B and C be subsets of the universal set U. Prove that: $(A \cup B) - C = (A - C) \cup (B - C)$.

<u>QUESTION 3</u> [11=3+8 marks]

1- Let A, B and C be subsets of the universal set U. Prove that : $A \times (B \cup C) = (A \times B) \cup (A \times C)$

2- Let R and S be two relations defined on the set $A = \{1,2,4\}$ as follows: $R = \{(x,y) \mid xy \text{ is even}\}$ and $S = \{(x,y) \mid x \text{ is a factor of } y\}$. Determine: (a) R and S (b) Dom(R) and Rng(S) (c) $S \circ R$ (d) $R^{-1} \circ S^{-1}$ (e) Which of R or S is antisymmetric?

QUESTION 4 [10=4+4+2 marks]:

1- Prove that $R = \{(x,y) \mid x + y \text{ is an even integer}\}$ is an equivalence relation on $\mathbb{Z}$ and find the distinct equivalence classes.

2- Let $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{2\}$ be a function defined as $f(x) = \dfrac{2x-1}{x-1}$. Prove that f is a one-to-one correspondence and find its inverse.

3- Let $f : A \to B$ and $g : B \to C$ be one-to-one functions. Prove that $g \circ f$ is also one-to-one function.

-Good Luck-