Kingdom of Saudi Arabia
Ministry of Education
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--- College of Science ---

بسم الله الرحمن الرحيم

المملكة العربية السعودية
وزارة التعليم
جامعة الإمام محمد بن سعود الإسلامية
كلية العلوم
قسم الرياضيات و الإحصاء

Department: Mathematics & Statistics
Semester/Year: First /1436-1437
Duration: 2 Hs

Course Name: Elements of sets and structures
Course Code: MAT 220

Final Examination

Answer Four questions only of the following:

QUESTION 1 [10=4+4+2 marks]

1. Let P, Q and R be statements. Use the stated laws and rules to prove that

$$((P \wedge Q) \Rightarrow R) \equiv (P \Rightarrow (Q \Rightarrow R)).$$

2. Prove, by the principle of mathematical induction, that:

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}, \quad \forall n \geq 1.$$

3. Write the converse, inverse, contrapositive and the negation of the following conditional statement:
" The number $\sqrt{2}$ is irrational, if the number $\log 2$ is rational ".

QUESTION 2 [10=4+4+2 marks]

1. Let the universe set be the set $U = \{1,2,3,\cdots,12\}$, $A = \{x \in U \,|\, x \leq 8\}$, $B = \{1,5,10\}$ and $C = \{x \in U \,|\, x \text{ is even}\}$. Determine: (a) $A' \cap C$ (b) $A - B$ (c) $|A \times B|$
(d) $P(B)$, the power set of the set B.

2. Let m and n be integers. Prove that if m is an even integer and n is an odd integer, then $\frac{m(n^2-1)}{8}$ is an even integer.

3. Find the truth set of $\{x \in \mathbb{N} \,|\, 2x + 5 \geq 3x\}$.

QUESTION 3 [10=3+(3+1+1)+2 marks]

1. Let A and B be subsets of the universal set U. Prove the following statement using the stated laws and rules: $(A \cup B) - C = (A - C) \cup (B - C)$.

"Please, turn over the page"

2. Let R and S be two relations defined on the set $A=\{a,b,c,d\}$ as follows
$R=\{(a,a),(b,b),(c,c),(d,d),(a,d),(d,a)\}$, $S=\{(a,a),(b,b),(c,d),(d,c),(c,c),(d,d)\}$.
Determine: (a) $S\circ R$. (b) Is $S\circ R$ an equivalence relation? (c) Is $R\cup S$ an equivalence relation? (Justify your answer).

3. Let R be a relation on a set A. Prove that $R=R^{-1}$ if and only if R is symmetric.

QUESTION 4 [**10=5+5 marks**]

1. Prove that $R=\{(x,y)\in\mathbb{Q}\times\mathbb{Q}\,|\,x-y \text{ is an integer}\}$ is an equivalence relation on $\mathbb{Q}$ and find the equivalence class $[0]_R$.

2. Prove that the function $f:\mathbb{R}-\{1\}\to\mathbb{R}-\{2\}$ defined by $f(x)=\dfrac{2x}{x-1}$ is a one-to-one correspondence and find f^{-1}.

QUESTION 5 [**10=5×2 marks**]

Prove or disprove **Five** of the following statements:

1- The statement $P\Rightarrow(P\vee Q)$ is a tautology.

2. Every relation is a function.

3. The integer 50 can be written as the sum of one odd integer and two even integers.

4. The function $f:[0,\infty)\to[1,\infty)$ defined by $f(x)=x^2+1$ is onto.

5. $R=\{(x,y)\in\mathbb{Z}\times\mathbb{Z}\,|\,x^2+y^2 \text{ is an even integer}\}$ is an equivalence relation on $\mathbb{Z}$.

Let A and B be subsets of a universal set U :

6. $P(A\cup B)=P(A)\cup P(B)$.

7. *If* $A\times B=B\times A$, *then* $A=B$.

************Best wishes************