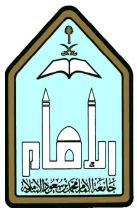
بسم اللّه الرَّحمَن الرَّحِيم

Kingdom of Saudi Arabia
Ministry of Higher Education
Al-Imam Mohammed Ibn Saud Islamic University
- College of Science -
Department: Maths & Stat.
1^{st} Semester/Year: 1436-1437
Duration: 2 hours



المَملَكَة العَرَبِية السَعُودِية
وزَارَة التَّعلِيم العَـــــالِي
جَامِعَة الإِمَام مُحَمَّد بن سَعُود الإِسلَامِية
-كلِّيـــــة العُـــــلوم-
Course Name: Cal. 2
Course Code: MAT 114
Coordinator:

Final Exam

Question 1. **[2+2+2+2+2+2 marks]**

1) Give the derivative of the function $f(x) = \int_1^{\ln(x^2+3)} \sqrt{t+1}\, dt.$

2) Determine the area of the region bounded by $y = \sqrt{x}, y = x^2, 1 \leq x \leq 2.$

3) Evaluate the following integrals

$$(a) \int_0^{\frac{\pi}{2}} \cos^3 x\, dx; \quad (b) \int (2x+1)\ln(x+1)\, dx;$$

$$(c) \int \frac{(\sqrt{x}+1)^2}{\sqrt{x}}\, dx; \quad (d) \int_0^{\infty} xe^{-x^2+3}\, dx.$$

Question 2. **[2+2+2+3 marks]**

1) Determine whether the following series converge or diverge.

$$(a) \sum_{k=1}^{\infty} \left[\frac{3}{3^k} + k^{-3}\right]; \quad (b) \sum_{k=1}^{\infty} (-1)^k \frac{5^k}{k!}; \quad (c) \sum_{k=1}^{\infty} \frac{2k^2+k}{2k^3-1}.$$

2) Determine the interval of convergence for the series

$$\sum_{k=0}^{\infty} \frac{(x+2)^k}{(2k+1)3^k}.$$

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Question 3. [2+2+2 marks]

Verify whether the following limits exist or not.

$$(a) \lim_{(x,y)\to(1,2)} \frac{x-1+y(x-1)}{(x-1)y}; \quad (b) \lim_{(x,y)\to(0,0)} \frac{xy^2}{-x^3-2y^3};$$

$$(c) \lim_{(x,y)\to(0,0)} \frac{y}{x^5-y}.$$

Question 4. [2+2+2+3+2+2 marks]

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \frac{1}{x} - xy^2.$$

1) Find the domain of the function f.

2) Evaluate the first order partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

3) Find the equation of tangent plane to the surface $z = f(x,y)$ at the point $(-1,1)$.

4) Evaluate the second order partial derivatives $\dfrac{\partial^2 f}{\partial x^2}$, $\dfrac{\partial^2 f}{\partial y^2}$ and $\dfrac{\partial^2 f}{\partial x \partial y}$.

5) Evaluate the integrals

$$(a) \int_1^2\int_1^2 f(x,y)\,dxdy; \quad (b) \int_1^2\int_1^x f(x,y)\,dxdy.$$