Kingdom of Saudi Arabia
Ministry of Education
Al-Imam Mohammed Ibn Saud
Islamic University
College of Science

Semester 2 - 1438/1437

المملكة العربية السعودية
وزارة التعليم
جامعة الإمام محمد بن سعود الإسلامية
-كلية العلوم-

Sun. 18/08/1438
Duration: 2H 30Min

Final Exam

CALCULUS II
MAT 106

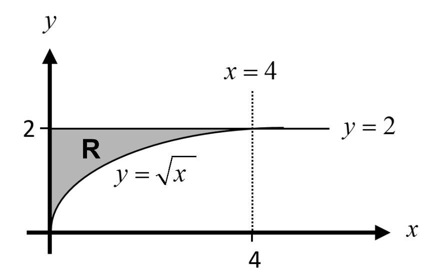
Section:______________ Student Name ____

Answers written outside the allocated space will NOT be graded. **Calculators are not allowed.**

Question:	1	2	3	4	Total
Points:	15	7	6	12	40
Score:					

1. $\boxed{\text{15 points}}$

(a) $\boxed{\text{3 points}}$ Let R be the region bounded by $y = \sqrt{x}$, the y-axis, and the $y = 2$, as shown in the figure. Find the volume of the solid resulting from revolving the region R about the x-axis.



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(b) $\boxed{\text{12 points}}$ Evaluate the following integrals:

i. $\displaystyle\int x\sec^2 x\; dx.$

ii. $\displaystyle\int \frac{x^3}{\sqrt{x^2+9}}\; dx.$

iii. $\displaystyle\int_0^2 \frac{e^x}{\sqrt{e^x - 1}}\, dx.$

iv. $\displaystyle\int_0^1 \int_0^{\sqrt{x}} 2\sqrt{x}\, e^{x^2}\, dy\, dx.$

2. $\boxed{\text{7 points}}$

(a) $\boxed{\text{4 points}}$ Determine whether the following series converges or diverges:

i. $$\sum_{k=3}^{\infty} \frac{(-3k)^k (k+1)^k}{k^{2k}}.$$

ii. $$\sum_{k=1}^{\infty} \frac{k^{-2}}{2+\sin^2 k}.$$

(b) $\boxed{\text{3 points}}$ Determine the interval and radius of convergence of the following power series:

$$\sum_{k=1}^{\infty} \frac{(x-4)^k}{\sqrt[3]{k}}$$

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3. $\boxed{\text{6 points}}$

(a) $\boxed{\text{2 points}}$ Find all polar coordinate representation for the rectangular coordinate $\left(-1, \sqrt{3}\right)$.

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(b) $\boxed{\text{2 points}}$ Find the slope of the tangent line to the polar curve $r = 3\sin\theta$ at $\theta = \dfrac{\pi}{4}$.

(c) $\boxed{\text{2 points}}$ Show that the rectangular equation $x^2 - 3x + y^2 = 0$ is corresponding to the polar equation $r = 3\cos\theta$.

4. $\boxed{\text{12 points}}$

(a) $\boxed{\text{3 points}}$ Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{3\cdot\sqrt{x^4+y^4}} = 0.$

(b) $\boxed{\text{3 points}}$ Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{3x^4y}{(2x^2+y)^3}$ does not exist.

(c) $\boxed{\text{3 points}}$ Let $f(x, y) = \sin(xy) + y^2 e^x$, compute f_{yx}.

(d) $\boxed{\text{3 points}}$ Let $f(x, y) = \ln\left(x^2 + y^2\right)$, show that $f_{xx} + f_{yy} = 0$.