



Final Examination

Question 1. (2+2+3+2 points):

Evaluate the following integrals

$$i) \int \frac{\sqrt{x^2 - 4}}{x} dx, \quad ii) \int \cos^3 x \sin^3 x dx \quad iii) \int_{-\infty}^0 \frac{e^x}{1 + e^x} dx, \quad iv) \int \frac{\ln x}{x^3} dx$$

Question 2. (3+3+2+4 points):

(1) Investigate the convergence or divergence of the following series:

$$i) \sum_{k=3}^{\infty} \frac{1}{k(\ln k)^2}, \quad ii) \sum_{k=0}^{\infty} (-1)^k \left(\frac{2k+1}{3k+1} \right)^k, \quad iii) \sum_{k=1}^{\infty} \frac{k^2 + 10k + 20}{k^3 + 10k + 10}$$

(2) Determine the interval and radius of convergence for the power series

$$\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2 2^k}.$$

Question 3. (3+4+4 points):

(1) Show that the following limit does not exist: $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2y}{x^4 + y^2}$

(2) For $f(x, y) = \cos(xy) - x^3y^4$, show that $f_{xy} = f_{yx}$.

Please turn over

(3) Compute the directional derivative $D_u f(3, -4)$ for the function $f(x, y) = \sqrt{x^2 + y^2}$ in the direction of the vector $u = \langle 3, -2 \rangle$.

Question 4. (4+4 points):

(1) Evaluate $\iint_R 2xy \, dA$, where R is the region bounded by the graphs of $y = x^2$ and $x + y = 2$.

(2) Evaluate the following integral by changing the order of integration: $\int_0^1 \int_{y^2}^1 2ye^{x^2} \, dx \, dy$.

Good Luck