

Quiz 1

Name:

ID:

Chose the correct answer:

1- If A is a 2x2 matrix with $|A| = -1$, then $\left|\left(\frac{1}{2}A\right)^{-1}\right|$ equal

- (a) 1 (b) -1 (c) -4 (d) 4

$$\left|\left(\frac{1}{2}A\right)^{-1}\right| = \left|\frac{1}{\frac{1}{2}} A^{-1}\right| = |2A^{-1}| = 2^2 \cdot \frac{1}{|A|} = -4$$

2- $\begin{vmatrix} ca & cb & -c^2 \\ ba & -b^2 & bc \\ -a^2 & ab & ac \end{vmatrix}$ equal

- (a) $-a^2b^2c^2$ (b) $-2a^2b^2c^2$ (c) $-4a^2b^2c^2$ (d) $-8a^2b^2c^2$

$$\begin{vmatrix} ca & cb & -c^2 \\ ba & -b^2 & bc \\ -a^2 & ab & ac \end{vmatrix} = abc \begin{vmatrix} a & b & -c \\ a & -b & c \\ -a & b & c \end{vmatrix} \begin{matrix} R_2+R_1 \\ R_3+R_1 \end{matrix} = abc \begin{vmatrix} a & b & -c \\ 2a & 0 & 0 \\ 0 & 2b & 0 \end{vmatrix}$$

$$= abc(-c)(4ba) = -4a^2b^2c^2$$

3- If $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$, then $(Adj(A))^{-1}$ is equal

- (a) $\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 2 \\ 4 & -3 \end{bmatrix}$

We have $A^{-1} = \frac{1}{|A|} Adj(A)$.

$\therefore |A| = 1 \Rightarrow A^{-1} = Adj(A) \Rightarrow (A^{-1})^{-1} = (Adj(A))^{-1}$

That is $(Adj(A))^{-1} = A$

4- If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ with $|A| = 5$, then $\begin{bmatrix} 2a & b & 2c \\ d & \frac{e}{2} & f \\ 2g & h & 2i \end{bmatrix}$ equal

- (a) 0 (b) 10 (c) 20 (d) 40

Let $\Delta = \begin{vmatrix} 2a & b & 2c \\ d & \frac{e}{2} & f \\ 2g & h & 2i \end{vmatrix} \Rightarrow 2\Delta = \begin{vmatrix} 2a & b & 2c \\ 2d & e & 2f \\ 2g & h & 2i \end{vmatrix} \Rightarrow 4\Delta = \begin{vmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{vmatrix} =$
 $= |2A| = 2^3 |A| \Rightarrow \Delta = 2 |A| = 10$

5- For which values of k does $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ k & 3 & 2 \end{bmatrix}$ fail to be invertible?

- (a) 0 (b) -1 (c) 1 (d) 4

The matrix A fails to be invertible if $|A| = 0$

$$\Rightarrow 1(2 - 18) - 2(6 - 6k) + 4(9 - k) = 0$$

$$\Rightarrow -16 - 12 + 12k + 36 - 4k = 0$$

$$\Rightarrow 8k + 8 = 0 \Rightarrow k = -1$$