Quiz 1

Name: ID:

Chose the correct answer:

1- If A is a 2x2 matrix with $|A| = -\frac{1}{4}$, then $|(-2A)^{-1}|$ equal

(a) 1 (b) -1 (c) -4 (d) 4

$$|(-2A)^{-1}| = |\tfrac{1}{-2}A^{-1}| = (-\tfrac{1}{2})^2 \cdot \frac{1}{|A|} = \frac{1 \cdot 4}{4 \cdot (-1)} = -1$$

2- $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$ equal

(a) $a^2b^2c^2$ (b) $2a^2b^2c^2$ (c) $4a^2b^2c^2$ (d) $8a^2b^2c^2$

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -a^2 \end{vmatrix} = abc \begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix} \overset{C_2+C_1,\ C_3+C_1}{=} abc \begin{vmatrix} -a & 0 & 0 \\ b & 0 & 2b \\ c & 2c & 0 \end{vmatrix}$$

$$= abc(-a)(-4bc) = 4a^2b^2c^2$$

3-If $A = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}$, then $(Adj(A))^{-1}$ is equal

(a) $\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 2 \\ 4 & -3 \end{bmatrix}$

We have $A^{-1} = \frac{1}{|A|} Adj(A)$

$\because |A| = 1 \Rightarrow A^{-1} = Adj(A) \Rightarrow (A^{-1})^{-1} = (Adj(A))^{-1}$

That is $(Adj(A))^{-1} = A$

4- If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ with $|A| = 10$, then $\begin{vmatrix} 2a & b & 2c \\ d & \frac{e}{2} & f \\ 2g & h & 2i \end{vmatrix}$ equal

(a) 5 (b) 10 (c) 20 (d) 40

Let $\Delta = \begin{vmatrix} 2a & b & 2c \\ d & \frac{e}{2} & f \\ 2g & h & 2i \end{vmatrix} \Rightarrow 2\Delta = \begin{vmatrix} 2a & b & 2c \\ 2d & e & 2f \\ 2g & h & 2i \end{vmatrix} \Rightarrow 4\Delta = \begin{vmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{vmatrix}$

$= |2A| = 2^3|A| \Rightarrow \Delta = 2|A| = 20$

5- For which values of k does $A = \begin{bmatrix} 3 & 1 & 6 \\ 1 & 2 & 4 \\ k & 3 & 2 \end{bmatrix}$ fail to be invertible?

(a) 0 (b) -1 (c) 1 (d) 4

The matrix A fails to be invertible if $|A| = 0$

$\Rightarrow 3(4-12) - 1(2-4k) + 6(3-2k) = 0$

$\Rightarrow -24 - 2 + 4k + 18 - 12k = 0$

$\Rightarrow -8 - 8k = 0 \Rightarrow k = -1$.