KINGDOM OF SAUDI ARABIA

Ministry of Higher Education

Imam Mohammad Ibn Saud Islamic University

College of Science

Department of Mathematics & Statistics

1434/1435 Second Semester

Course Code &Number: Mat 321

Course Title: Modern Algebra

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Midterm 1

Duration: 1H15

(We will write a instead of $[a]$ in $\mathbb{Z}_n$)

Solution Q1: The subgroups are: $\langle 1\rangle, \langle 2\rangle, \langle 3\rangle, \langle 5\rangle, \langle 6\rangle, \langle 10\rangle, \langle 15\rangle, \langle 0\rangle$, and the generators are: $1, 7, 11, 13, 17, 19, 23, 29$.

Solution Q2: Let $G = \langle a\rangle$ be cyclic group, and $x, y \in G$, then $\exists m, n \in \mathbb{Z} : x = a^m, y = a^n$. Now

$xy = a^m a^n = a^{m+n} = a^{n+m} = a^n a^m = yx$. That is G is Abelian.

As $\langle 3\rangle = \{1, 3, 7, 9\} = \langle 7\rangle$, then G is cyclic with generator 3 or 7.

Solution Q3: $\langle 3\rangle = \{1, 3, 4, 5, 9\}$. As the inverse of 4 is 3 in $\mathbb{Z}_{11}^*$, then $O(4) = O(3) = |\langle 3\rangle| = 5$. Let G be a group of even order. We know that the number of elements of order 2 is odd and $O(e) = 1$. That is the number of all elements of order less than or equal 2 is even. So the number of elements of order greater than 2 is even. So the statement is wrong.

Solution Q4: $\varphi(xy) = \frac{|xy|}{xy} = \frac{|x|}{x}\frac{|y|}{y} = \varphi(y)\varphi(y)$. Then φ is a homomorphism and as it is from $\mathbb{R}^*$ to itself, then it is an endomorphism. $Ker\varphi = \mathbb{R}^+$, $\operatorname{Im}\varphi = \varphi(\mathbb{R}^*) = \{1, -1\}$. $\varphi(3) = 6$, since the order of the element 3 in $\mathbb{Z}_6$ under addition is 2. So its image must be of order 2 also and the unique element of order 2 in $\mathbb{Z}_7^*$ under multiplication is the element 6.

Solution Q5: $f(x+y) = 4(x+y) = 4x + 4y = f(x) + f(y)$. Then f is a homomorphism. $Kerf = \{0, 5, 10, 15\}$. So f is not monomorphism, because $Kerf \neq \{0\}$. $\operatorname{Im} f = f(\mathbb{Z}_{20}) = \{0, 4, 8, 12, 16\}$. So f is not epimorphism, because $\operatorname{Im} f \neq \mathbb{Z}_{20}$ (The codomain)

Solution Q6: $\alpha = \begin{pmatrix}1 & 7 & 6 & 4 & 3 & 2\end{pmatrix}$. $O(\alpha) = 6$.

$\alpha = \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}1 & 6\end{pmatrix}\begin{pmatrix}1 & 7\end{pmatrix}$ and α is an odd permutation.

$Z(S_3) = \{e\}$, since

$\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}1 & 3\end{pmatrix} \neq \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}$

$\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}2 & 3\end{pmatrix} \neq \begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}$

$\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}1 & 2 & 3\end{pmatrix} \neq \begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}$

$\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}1 & 3 & 2\end{pmatrix} \neq \begin{pmatrix}1 & 3 & 2\end{pmatrix}\begin{pmatrix}1 & 2\end{pmatrix}$

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