Kingdom of Saudi Arabia Ministry of Education Al-Imam Mohammed Ibn Saud Islamic University College of Science	Semester: 143820	المملكة العربية السعودية وزارة التعليم جامعة الإمام محمد بن سعود الإسلامية -كليـــة العـــلوم-

MAT 106 **MIDTERM 1** **CALCULUS II**

Duration: 1H 15Min Section:________ Student Name ________________

Answers written outside the allocated space will NOT be graded. **Calculators are not allowed.**

Question:	1	2	3	Total
Points:	4	6	10	20
Score:				

1. $\boxed{\text{4 points}}$ Evaluate the following definite integrals:

(a) $\displaystyle\int_1^2 \frac{\ln(x)}{x^2}\,dx.$

$u = \ln(x)$ $dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx$

$v = \frac{-1}{x}$

$\displaystyle\int_1^2 \frac{\ln(x)}{x^2}\,dx = \frac{-\ln x}{x}\Big|_1^2 + \int_1^2 \frac{1}{x^2}\,dx$

$= \left(\frac{-\ln x}{x} - \frac{1}{x}\right)\Big|_1^2 = \left(\frac{-\ln 2}{2} - \frac{1}{2}\right) - \left(\cancel{\frac{-\ln 1}{1}} - \frac{1}{1}\right) = \boxed{\frac{-\ln 2}{2} + \frac{1}{2}}$

0

(b) $\displaystyle\int_0^4 x\sqrt{x^2+9}\,dx.$

$u = x^2 + 9$

$du = 2x\,dx \Rightarrow dx = \frac{du}{2x}$

$\displaystyle\int \cancel{x}\sqrt{u}\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int u^{1/2}\,du$

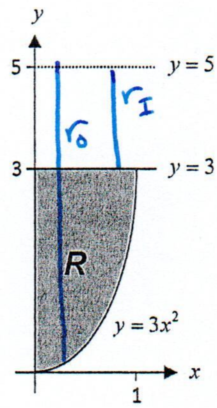
$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} = \frac{1}{3}\left(\sqrt{x^2+9}\right)^3\Big|_0^4$

$= \left(\frac{1}{3}\left(\sqrt{16+9}\right)^3\right) - \left(\frac{1}{3}\left(\sqrt{0+9}\right)^3\right)$

$= \frac{5^3}{3} - \frac{3^3}{3} = \frac{125}{3} - \frac{27}{3}$

$= \boxed{\frac{98}{3}}$

2. $\boxed{\text{6 points}}$ Let R be the region bounded by $y = 3x^2$, the y-axis, and the $y = 3$, as shown in the figure. Find the volume of the solid resulting from revolving the region R about:



(a) the y-axis.

$$V = \int_0^3 \pi \left(g(y)\right)^2 dy$$

$$y = 3x^2 \Rightarrow x^2 = \frac{y}{3} \Rightarrow x = g(y) = \sqrt{\frac{y}{3}}$$

$$V = \int_0^3 \pi \left(\sqrt{\frac{y}{3}}\right)^2 dy = \frac{\pi}{3}\int_0^3 y\,dy = \frac{\pi\, y^2}{6}\Big|_0^3$$

$$= \frac{\pi}{6}(9 - 0) = \boxed{\frac{3\pi}{2}}$$

(b) the line $y = 5$.

$$V = V_o - V_I$$

$$r_o = 5 - 3x^2$$
$$r_I = 5 - 3 = 2$$

$$= \int_0^1 \pi (r_o)^2 dx - \int_0^1 \pi (r_I)^2 dx$$

$$= \int_0^1 \pi (5 - 3x^2)^2 dx - \int_0^1 \pi (2)^2 dx$$

$$= \int_0^1 \pi (25 - 30x^2 + 9x^4)\, dx - \int_0^1 4\pi\, dx$$

$$= \pi\left(25x - 10x^3 + \tfrac{9}{5}x^5\right)\Big|_0^1 - (4\pi x)\Big|_0^1$$

$$= \left[\pi\left(25 - 10 + \tfrac{9}{5}\right) - \pi(0)\right] - \left[4\pi - 0\right]$$

$$= \pi\left(15 + \tfrac{9}{5}\right) - 4\pi = \boxed{\frac{64}{5}\pi}$$

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3. $\boxed{\text{10 points}}$ Evaluate the following integrals:

(a) $\displaystyle\int 3x^2 e^x \, dx.$

$3x^2$		e^x
	$+$	
$6x$		e^x
	$-$	
6		e^x
	$+$	
0		e^x

$$\int 3x^2 e^x \, dx = \boxed{3x^2 e^x - 6x e^x + 6 e^x + C}$$

(b) $\displaystyle\int \frac{4x}{x^2 + 2x + 5} \, dx.$

$$= 2 \int \frac{2x + 2 - 2}{x^2 + 2x + 5} \, dx$$

$$= 2 \left[\int \frac{2x+2}{x^2 + 2x + 5} \, dx - \int \frac{2}{x^2 + 2x + 5} \, dx \right]$$

$$= 2 \left[\ln|x^2 + 2x + 5| - 2 \int \frac{1}{x^2 + 2x + 1 - 1 + 5} \, dx \right]$$

$$= 2 \left[\ln|x^2 + 2x + 5| - 2 \int \frac{1}{(x+1)^2 + 4} \, dx \right]$$

$$= 2 \left[\ln|x^2 + 2x + 5| - \frac{2}{2} \tan^{-1}\left(\frac{x+1}{2}\right) \right] + C$$

$$= \boxed{2 \ln|x^2 + 2x + 5| - 2 \tan^{-1}\left(\frac{x+1}{2}\right) + C}$$

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(c) $\int \cos^7 x \cdot \sin^3 x \, dx.$

$u = \cos x$

$du = -\sin x \, dx \Rightarrow dx = \frac{-du}{\sin x}$

$\int u^7 \sin^{\cancel{3}\,2} x \, \frac{-du}{\cancel{\sin x}} = -\int u^7 (1 - \cos^2 x) \, du$

$= -\int u^7 (1 - u^2) \, du = -\int u^7 - u^9 \, du$

$= -\left(\frac{u^8}{8} - \frac{u^{10}}{10} \right) + C$

$= -\left(\frac{\cos^8 x}{8} - \frac{\cos^{10} x}{10} \right) + C = \boxed{\frac{\cos^{10} x}{10} - \frac{\cos^8 x}{8} + C}$

(d) $\int \sqrt{\tan x} \cdot \sec^4 x \, dx.$

$u = \tan x$

$du = \sec^2 x \, dx \Rightarrow dx = \frac{du}{\sec^2 x}$

$\int \sqrt{u} \sec^{\cancel{4}\,2} x \, \frac{du}{\cancel{\sec^2 x}} = \int \sqrt{u} \, (\tan^2 x + 1) \, du$

$= \int u^{1/2} (u^2 + 1) \, du = \int u^{5/2} + u^{1/2} \, du$

$= \frac{u^{7/2}}{7/2} + \frac{u^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{7} \tan^{7/2} x + \frac{2}{3} \tan^{3/2} x + C}$

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(e) $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx.$

$x = 4\sin\theta \Rightarrow dx = 4\cos\theta\, d\theta$

$$\int \frac{x^2}{\sqrt{16-x^2}}\,dx = \int \frac{16\sin^2\theta}{\sqrt{16-16\sin^2\theta}}\, 4\cos\theta\, d\theta$$

$$= \int \frac{16\sin^2\theta}{\cancel{4}\,\cancel{\cos}\,\theta}\, \cancel{4}\,\cancel{\cos}\theta\, d\theta$$

$$= 16\int \frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= 8\int 1-\cos 2\theta\, d\theta$$

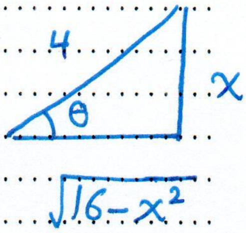
$$= 8\left(\theta - \frac{\sin 2\theta}{2}\right) + C$$

$$= 8\left(\theta - \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right) + C$$

$$= 8(\theta - \sin\theta\cos\theta) + C$$

$$= 8\left(\sin^{-1}\frac{x}{4} - \frac{x}{4}\cdot\frac{\sqrt{16-x^2}}{4}\right) + C$$

$$= \boxed{8\sin^{-1}\frac{x}{4} - \frac{x\sqrt{16-x^2}}{2} + C}$$



$\sin\theta = \dfrac{x}{4}$

$\theta = \sin^{-1}\dfrac{x}{4}$

$\cos\theta = \dfrac{\sqrt{16-x^2}}{4}$

Good Luck

THE END