MIDTERM (1)



Kingdom of Saudi Arabia AL-Imam Mohammed Bin Saud

 ${\bf Islamic\ University}$

College of Science

Department of Mathematics

Course name: Calculus II

Course code: MAT 102

Semester: 1st /1437 -1438

Duration: 1:30

Dr. Ghaliah Alhamzi

Name	
Student Number	
Section	

Question's number	Marks
1	
2	
3	
TOTAL	

Question 1

(a) Give the definition of definite integral and evaluate the following integrals

Definition: (1 Mark)

(i)
$$\int \left(3\sqrt{x} - \frac{1}{x^4}\right) dx$$
 (1 Mark)

(ii)
$$\int (3\tan x + 4)^5 \sec^2 x \, dx$$
 (2 Mark)

(b) Compute the integral $\int e^x \sin 4x \, dx$ (2 Mark)

Question 2

(a) Evaluate
$$\int \cos^4 x \sin^3 x \, dx$$
 (2 Mark)

(b) Evaluate each of the following.

(i)
$$\lim_{x \to \infty} \frac{x^2}{e^x}$$
 (2 Mark)

(ii)
$$\lim_{x \to 0^+} \left(\frac{1}{\sin x} - \cot x \right)$$
 (2 Mark)

(c) Calculate the integral

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} \, dx$$

(2 Mark)

Question 3

(a) Use the Squeeze Theorem to prove that the given sequence converges to 0

$$a_n = \frac{\cos n}{n^2} \ .$$

(2 Mark)

(b) Determine whether the series converges or diverges. For convergent series, find the sum of the series.

(i)
$$\sum_{k=0}^{\infty} \left(\frac{1}{3}\right) 5^k \tag{2 Mark}$$

(ii)
$$\sum_{k=1}^{\infty} \frac{4}{k(k+1)}$$
 (2 Mark)