MIDTERM (2)

Kingdom of Saudi Arabia
AL-Imam Mohammed Bin Saud
Islamic University
College of Science
Department of Mathematics

Course: Calculus III
Course code: MAT 203
Semester: 1st /1438
Duration: 1Hour

Dr. Ghaliah Alhamzi

Name	
Student Number	
Section	

Question's number	Marks
1	/10
2	/10
TOTAL	/ 20

Question 1

(i) Show $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+4x^2+2y^2}{2x^2+y^2}$ exists. (2 Marks)

(ii) Let $f(x,y) = e^{xy}+\ln(x^2+y)$, show that $f_{xy} = f_{yx}$. (3 Marks)

(iii) Find equations of the tangent plane and the normal line to

$$z = 6 - x^2 - y^2 \quad \text{at the point } (1,2,1).$$

(2 Marks)

(v) Consider the function

$$z = f(x,y) = \sin(x+y) \quad \text{with} \quad x = uv^2 \quad \text{and} \quad y = u^2 + \frac{1}{v},$$

find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$. (3 Marks)

Question 2

(i) For $f(x,y) = x^2 + y^2$,
compute $D_{\hat{u}}f(1,-1)$ for $\hat{u}$ in the direction of $\overrightarrow{v} = \langle -3, 4\rangle$ (2 Marks)

(ii) locate all critical points and classify them using (Second Derivatives Test)

$$f(x,y) = e^{-x^2}(y^2+1)$$

(3 Marks)

(iii) Find the volume beneath the surface and above the rectangular region

$$z = x^2 + y^2, \quad 0 \le x \le 3, \quad 1 \le y \le 4$$

(3 Marks)

(v) Change the order of integration

$$\int_{x=0}^{x=1} \int_{y=0}^{y=2x} f(x,y)\, dy\, dx$$

(2 Marks)