Experiment 4

Damped and Forced Oscillations – Pohl's Torsional Pendulum

1- Objects of the experiment

- Determine the oscillating period and the characteristic frequency of the undamped case.
- Determine the oscillating periods and the corresponding characteristic frequencies for different damping values. The corresponding damping constants and the logarithmic decrements are to be calculated.
- Determine the resonance curves in the case of forced oscillations and represent them graphically using the damping values. Determine the corresponding resonance frequencies and compare them with the resonance frequency values found beforehand.

2- Principles

2-1- Rotating pendulum with undamped and free oscillations:

When the object (copper disk) is twisted through some angle ϕ, the twisted spiral spring exerts on the object a ***restoring torque*** that is proportional to the angular position. That is

$$\tau_r = -k\phi \qquad (1)$$

where k is called the ***torsion constant*** of the spiral spring.

The equation of motion of the system can be derived by applying Newton's second law for rotational motion, we find that

$$\sum \tau = \tau_r = I\frac{d^2\phi}{dt^2} \qquad (2)$$

where I is the moment of inertia of copper disk

$$\frac{d^2\phi}{dt^2} + \frac{k}{I}\phi = 0 \qquad (3)$$

This result is the equation of motion for a simple harmonic oscillator, with the *natural frequency of the undamped system*

$$\omega_r = \sqrt{\frac{k}{I}} \qquad (4)$$

and a period

$$T_r = 2\pi\sqrt{\frac{I}{k}} \qquad (5)$$

2-2- Rotating pendulum with damped and free oscillations:

In this case, i.e. without external drive, the system can be modeled in the following way: if the pendulum is deflected by an angle ϕ and released, it will perform a damped oscillation around the equilibrium angle (i.e., the mechanical energy of the system diminishes in time). This oscillation is called *damped natural oscillation of the system*. The equation of motion of the system can be derived by applying Newton's second law for rotational motion, we find that

$$\sum\tau = \tau_r + \tau_d = I\frac{d^2\phi}{dt^2} \qquad (6)$$

where $\tau_d = -b\frac{d\phi}{dt}$ is the damping torque due to the Eddy current break with b the damping coefficient.

Equation (6) leads to the differential equation of the damped natural oscillation of the pendulum:

$$I\frac{d^2\phi}{dt^2} + b\frac{d\phi}{dt} + k\phi = 0 \qquad (7\text{-a})$$

$$\frac{d^2\phi}{dt^2} + \frac{b}{I}\frac{d\phi}{dt} + \frac{k}{I}\phi = 0 \qquad (7\text{-b})$$

$$\frac{d^2\phi}{dt^2} + 2\delta\frac{d\phi}{dt} + \omega_r^2\phi = 0 \qquad (7\text{-c})$$

where $\delta = \frac{b}{2I}$ is called the *damping constant*

If the oscillation begins with maximum amplitude ϕ_0 at t=0, the resulting solution to the differential equation (7-c) for light damping ($\delta^2\langle\omega_r^2$) is as follows

$$\phi(t) = \phi_0 e^{-\delta t}\cos\omega_d t \qquad (8)$$

with

$$\omega_d = \sqrt{\omega_r^2 - \delta^2} \qquad (9)$$

is the *natural frequency of the damped system*.

Under heavy damping ($\delta^2\rangle\omega_r^2$) the system does not oscillate but moves directly into a state of rest or equilibrium (non-oscillating case).

The period duration $T_d = \frac{2\pi}{\omega_d}$ of the lightly damped oscillating system varies only slightly from T_r of the undamped oscillating system.

By inserting $t = nT_d$ into the equation (8) then $\phi(nT_D) = \phi_n = \phi_0 e^{-n\delta T_D}$ is the *amplitude after n periods.* We obtain the following equation:

$$\frac{\phi_n}{\phi_0} = e^{-n\delta T_D} \qquad (10)$$

and thus from this the *logarithmic decrement* Λ is:

$$\Lambda = \delta T_d = \frac{1}{n}\ln\left(\frac{\phi_0}{\phi_n}\right) = \ln\left(\frac{\phi_n}{\phi_{n+1}}\right) \qquad (11)$$

2-3- Rotating pendulum with damped and forced oscillations:

In the case of forced oscillations, a rotating motion with sinusoidally varying torque is externally applied to the system. This exciter torque ($\tau_e = \hat{\tau}_e \sin(\omega_e t)$) can be incorporated into the motion equation as follows:

$$I \frac{d^2\phi}{dt^2} + b\frac{d\phi}{dt} + k\phi = \hat{\tau}_e \sin(\omega_e t) \tag{12}$$

After a transient or settling period the torsion pendulum oscillates in a steady state with the same angular frequency as the exciter. β is the system's zero-phase angle, the phase displacement between the oscillating system and the exciter.

$$\phi(t) = \phi_e \sin(\omega_e t - \beta) \tag{13}$$

The following hold true for the system's amplitude ϕ_e:

$$\phi_e(\omega_e) = \frac{\hat{\tau}_e / I}{\sqrt{\left(\omega_r^2 - \omega_e^2\right)^2 + 4\delta^2\omega_e^2}} \tag{14}$$

and the system's zero phase angle β:

$$\beta = \arctan\left(\frac{2\delta\omega_e}{\omega_r^2 - \omega_e^2}\right) \tag{15}$$

In the case of damped oscillations with light damping, the system amplitude reaches a maximum when the exciter's angular frequency ω_e is near the natural frequency of oscillation. The dramatic increase in amplitude near the natural frequency is called *resonance*. This frequency is given by

$$\omega_e = \omega_{resonance} = \omega_r \sqrt{1 - \frac{2\delta^2}{\omega_r^2}} \qquad \text{for } \phi_e \text{ maximum} \tag{16}$$

3- List of equipment

Apparatus	Quantity	Catalogue Number
Torsion pendulum after Pohl	1	11214.00
Bridge rectifier, 30 V AC/1 A DC	1	06031.10
Stopwatch, digital, 1/100 sec.	1	03071.01
Digital multimeter	2	07134.00
Variable transformer, 25 VAC/ 20 VDC, 12 A	1	13531-93
Connecting cord, l = 250 mm, yellow	2	07360.02
Connecting cord, l = 750 mm, red	2	07362.01
Connecting cord, l = 750 mm, blue	3	07362.04

4- Safety instructions, Description, and Technical data

1- When carrying the torsional pendulum always hold it by the base plate.

2- Never exceed the maximum permissible supply voltage for the exciter motor (24 V DC).

3- Do not subject the torsional pendulum to any unnecessary mechanical stress.

The Professor Pohl torsional pendulum consists of a wooden base plate with an oscillating system and an electric motor mounted on top. The oscillating system is a ball-bearing mounted copper wheel, which is connected to the exciter rod via a coil spring that provides the restoring torque. A DC motor with coarse and fine speed adjustment (Figure 5) is used to excite the torsional pendulum. Excitement is brought about via an eccentric wheel with connecting rod which unwinds the coil spring then compresses it again in a periodic sequence and thereby initiates the oscillation of the copper wheel. *The electromagnetic eddy current brake is used for damping*. A scale ring with slots and a scale in 2-mm divisions extends over the outside of the oscillating system; indicators are located on the exciter and resonator.

Motor: max. 24 V DC, 0.7 A, via 4-mm safety sockets

Eddy current brake: 0 to 24 V DC, max. 2 A, via 4-mm safety sockets

A DC power supply unit for the torsional pendulum is required to power the equipment. Do not extract the 4mm plugs from the sockets of the power supply unit during operations, because there is a danger of burns due to the formation of arcs and the output sockets can be damaged.

5- Setup and carrying out the experiment

The experiment is set up as shown in Figures 1, and 2. The DC output of the power supply unit is connected to the eddy current brake. The motor also needs DC voltage; for this reason a bridge rectifiers is inserted between the AC output (**12V**) of the power supply unit and to the two right sockets of the DC motor (see Figure 3). Before switching the power supply on, set adjusting knob 2 (see Figure 4) to the minimum value (left stop) and adjusting knob 1 to maximum voltage. The DC current supplied to the eddy current brake, I_B, is set with the adjusting knob 2 (of the power supply) and is indicated by the ammeter.

Figure 1. Experimental set-up.

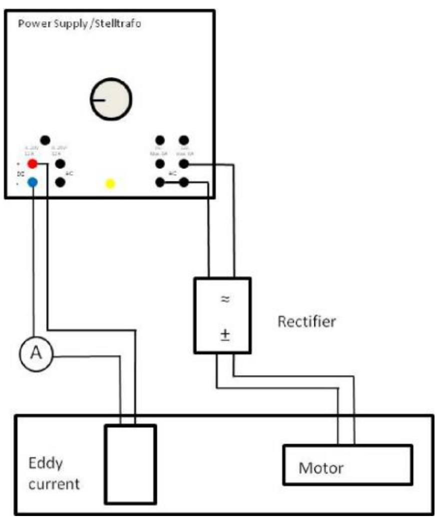

Figure 2. Electrical connection of the experiment.

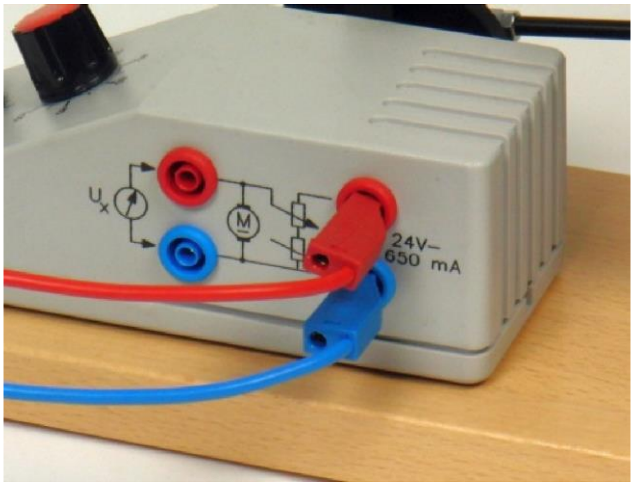

Figure 3. Connection of the DC motor to the power supply.

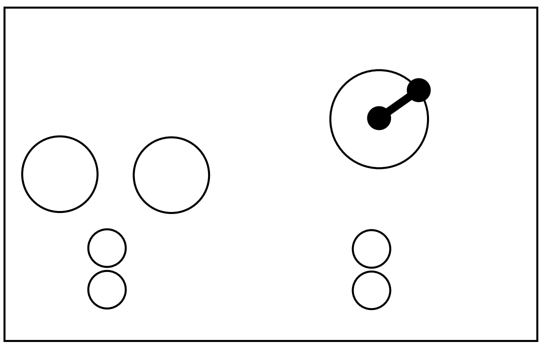

Figure 4. Some elements of the power supply unit.

Figure 5. Control knobs to set the motor potentiometer. Upper knob: “coarse”; lower knob: “fine”.

5-1- Rotating pendulum with undamped (I_B = 0) and free oscillations (τ_e = 0):

1) Disconnect the DC power to motor and Eddy current ($\tau_e = 0$, $I_B = 0$)

2) Release the disk with an angle (for example 15º) and measure the amplitude for each period n until n = 5. Repeat the measurements four times and calculate the average.

3) Record the measurements in Table 1.

4) Calculate the logarithmic decrements, Λ, for n = 1 to n = 5 by using Equation (11)

5) Calculate the average value of Λ

Table 1. Determination of amplitudes, ϕ_n, after n periods and calculation of logarithmic decrements Λ ($I_B = 0$ and $\tau_e = 0$)

n	ϕ_n				$\phi_{n,average}$	Λ
0	15	15	15	15	15	-----------
1	………..	………..	………..	………..		
2	………..	………..	………..	………..		
3	………..	………..	………..	………..		
4	………..	………..	………..	………..		
5	………..	………..	………..	………..		

6) To measure the pendulum's oscillation period, T_d, record the time for 10 oscillations using a stop watch by taking $\phi_0 = 15^{\circ}$. Repeat the measurements three times

7) Record the measurements in Table 2.

Table 2. Measurement of the pendulum's oscillation period ($I_B = 0$ and $\tau_e = 0$)

$10T_{d,1}$ (s)	$10T_{d,2}$ (s)	$10T_{d,3}$ (s)	$10T_{d,average}$ (s)	T_d (s)	ω_d (s^{-1})	δ (s^{-1})
……….	……….	……….				

8) Calculate the damping constant, δ, by using Equation (11) and record it in Table 2

5-2- Rotating pendulum with damped ($I_B \neq 0$) and free oscillations (τ_e = 0):

1) Connect the eddy current brake to the variable voltage output of the DC power supply for torsion pendulum and connect the ammeter into the circuit.
2) Repeat the steps from 2 to 8 as mentioned in the above section (5-1) for I_B = 0.2A, 0.4A, and 0.6A

Table 3. Determination of amplitudes, ϕ_n, after n periods and calculation of logarithmic decrements Λ (I_B = 0.2A and $\tau_e = 0$)

n	ϕ_n				$\phi_{n,average}$	Λ
0	15	15	15	15	15	-----------
1	………..	………..	………..	………..		
2	………..	………..	………..	………..		
3	………..	………..	………..	………..		
4	………..	………..	………..	………..		

5	………..	………..	………..	………..		

Table 4. Measurement of the pendulum's oscillation period (I_B = 0.2A and $\tau_e = 0$)

$5T_{d,1}$ (s)	$5T_{d,2}$ (s)	$5T_{d,3}$ (s)	$5T_{d,average}$ (s)	T_d (s)	ω_d (s^{-1})	δ (s^{-1})
……….	……….	……….				

Table 5. Determination of amplitudes, ϕ_n, after n periods and calculation of logarithmic decrements Λ (I_B = 0.4A and $\tau_e = 0$)

n	ϕ_n				$\phi_{n,average}$	Λ
0	15	15	15	15	15	-----------
1	………..	………..	………..	………..		
2	………..	………..	………..	………..		
3	………..	………..	………..	………..		
4	………..	………..	………..	………..		
5	………..	………..	………..	………..		

- Calculate $\Lambda_{average}$

Table 6. Measurement of the pendulum's oscillation period (I_B = 0.4A and $\tau_e = 0$)

$5T_{d,1}$ (s)	$5T_{d,2}$ (s)	$5T_{d,3}$ (s)	$5T_{d,average}$ (s)	T_d (s)	ω_d (s^{-1})	δ (s^{-1})

………	………	………				

Table 7. Determination of amplitudes, ϕ_n, after n periods and calculation of logarithmic decrements Λ (I_B = 0.6A and $\tau_e = 0$)

n	ϕ_n				$\phi_{n,average}$	Λ
0	15	15	15	15	15	-----------
1	………..	………..	………..	………..		
2	………..	………..	………..	………..		
3	………..	………..	………..	………..		
4	………..	………..	………..	………..		
5	………..	………..	………..	………..		

- Calculate $\Lambda_{average}$

Table 8. Measurement of the pendulum's oscillation period (I_B = 0.6A and $\tau_e = 0$)

$5T_{d,1}$ (s)	$5T_{d,2}$ (s)	$5T_{d,3}$ (s)	$5T_{d,average}$ (s)	T_d (s)	ω_d (s^{-1})	δ (s^{-1})
………	………	………				

5-3- Rotating pendulum with damped ($I_B \neq 0$) and forced oscillations ($\tau_e \neq 0$):

1) Connect the DC power supply to the torsion pendulum as shown in Figure 1 and record the measurement in Tables 9, 10, 11, and 12 by taking in

consideration the values of I_B respectively as $I_B = 0$, $I_B = 0.2A$, $I_B = 0.4A$, and $I_B = 0.6A$.

2) Plot a graph of amplitude ϕ_e (vertical axis) versus exciter's angular frequency ω_e (horizontal axis) for $I_B = 0$, $I_B = 0.2A$, $I_B = 0.4A$, and $I_B = 0.6A$ in one graph paper.

3) Find the angular frequency resonance from the graph. This should be equal to the undamped free oscillation angular frequency, ω_d, that is found in Table 2.

Table 9.

$I_B = 0$				
Motor Voltage (V)	$10T_e$ *(s)*	T_e *(s)*	ω_e (s^{-1})	ϕ_e
6.0	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
11	………..			………..

Table 10.

$I_B = 0.2A$				
Motor Voltage (V)	$10T_e$ (s)	T_e (s)	ω_e (s^{-1})	ϕ_e
6.0	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
11	………..			………..

Table 11.

$I_B = 0.4A$				
Motor Voltage (V)	$10T_e$ (s)	T_e (s)	ω_e (s^{-1})	ϕ_e
6.0	………..			………..
	………..			………..
	………..			………..
	………..			………..

	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
11	………..			………..

Table 12.

$I_B = 0.6A$				
Motor Voltage (V)	$10T_e$ (s)	T_e (s)	ω_e (s^{-1})	ϕ_e
6.0	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
	………..			………..
11	………..			………..

6) Conclusions

- Discuss your results.