Experiment 3

Laws of Gyroscopes / 3-axis gyroscope

1- Objects of the experiment

1- Determination of the momentum of inertia, I, of the gyroscope by measurement of the angular acceleration α .

2- Determination of the momentum of inertia by measurement of the gyroscope frequency, ω_r , and precession frequency ω_{pr} . Investigation of the relationship between precession frequency and gyroscope frequency, and its dependence from torque.

3- Investigation of the relationship between nutation frequency, ω_n and gyroscope frequency.

2- Principles

A **gyroscope** is a rigid body that rotates around an axis that is fixed at one point. If no external torque is applied, the axis of the gyroscope (being equivalent to the axis of its angular momentum) maintains its position in space. If, however, an external force is applied to the axis, then this torque effects a change in angular momentum. As result, the axis is laterally displaced. The gyroscope moves in direction perpendicular to both its own axis and to the acting force. This motion is called **precession**. If an impulse is applied to the axis of the gyroscope when it is spinning normally, the resulting torque causes an additional angular momentum and the gyroscope starts to wobble. This wobbling motion is called **nutation**. In general, both motions are superimposed on one another.

2-1- Determination of the momentum of inertia of the gyroscope disk

If the gyroscope disk is set to rotate by means of a falling mass m (Figure 1), the following relation is valid for the angular acceleration:

$$\frac{d\omega_r}{dt} = \alpha = \frac{\tau}{I} \qquad (1)$$

where ω_r is the angular velocity; α is the angular acceleration; I is the moment of inertia; and τ the torque.

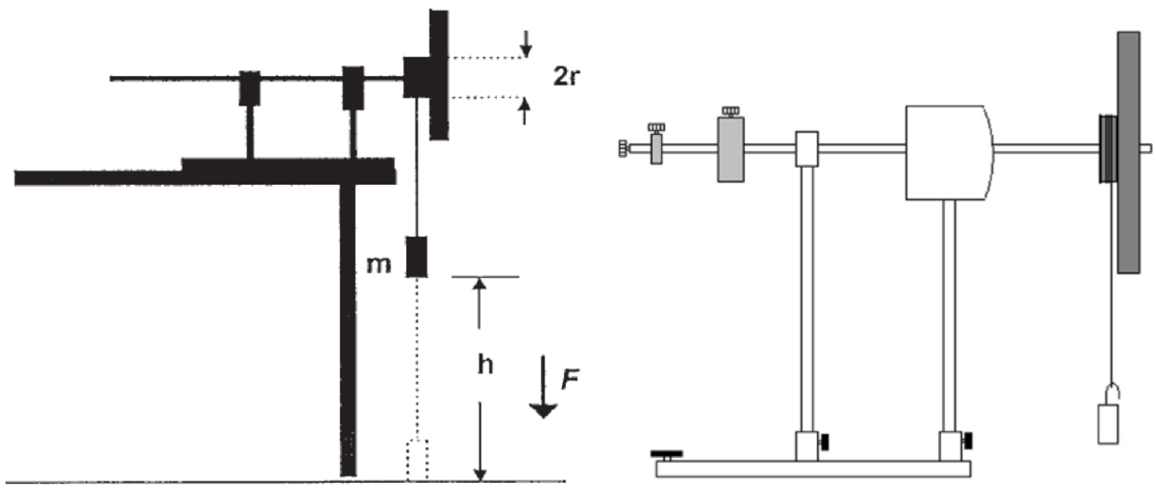
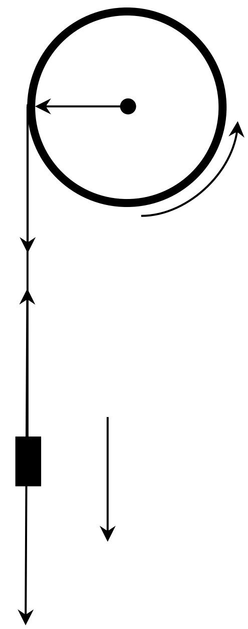


Figure 1. *Schematic representation of the experimental set-up to determine the moment of inertia of the gyroscope disk.*

The force, T, which causes the torque is given by the Newton's second law:

$$\vec{T} + m\vec{g} = m\vec{a}$$

$$T - mg = -ma \quad \Rightarrow \quad T = m(g-a) \qquad (2)$$



The following relations are true for the trajectory acceleration *a* and the angular acceleration α :

$$a = \frac{2h}{t_f^2} \qquad (3\text{-a})$$

$$\alpha = \frac{a}{r} \qquad (3\text{-b})$$

Where h is the dropping height of the acceleration mass, t_f is the falling time.

Introducing (2) and (3) into (1), one obtains

$$t_f^2 = \frac{2I + 2mr^2}{mgr^2}h \qquad (4)$$

2-2- Determination of the precession frequency

Let the symmetrical gyroscope in Figure 2, which is suspended so as to be able to rotate around the 3 main axes, be in equilibrium in horizontal position with counterweight C. If the gyroscope is set to rotate around the x-axis, with an angular velocity ω_r, the following is valid for the angular momentum L, which is constant in space and in time:

$$L = I\omega_r \qquad (5)$$

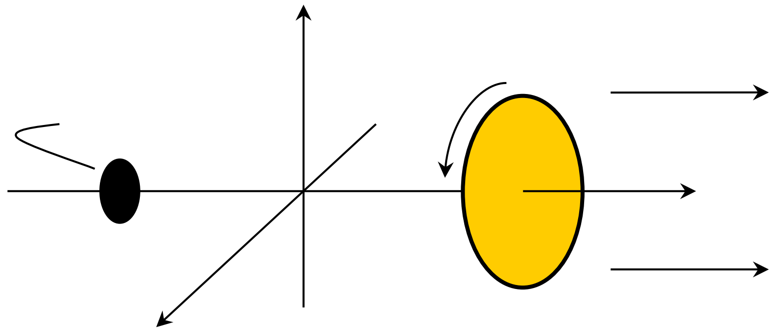


Figure 2. Schematic representation of the gyroscope submitted to no external torque

Adding a supplementary mass m^* at the distance r^* from the support point, O, induces a supplementary torque $\vec{\tau}^*$, which is equal to the variation in time of the angular momentum, and parallel to it.

$$\vec{\tau}^* = \frac{d\vec{L}}{dt} \qquad (6)$$

Besides

$$\vec{\tau}^* = \vec{r}^* \times m^* \vec{g} = r^* m^* g \vec{a}_z \qquad (7\text{-a})$$

Magnitude of the torque

$$\tau^* = r^* m^* g \qquad (7\text{-b})$$

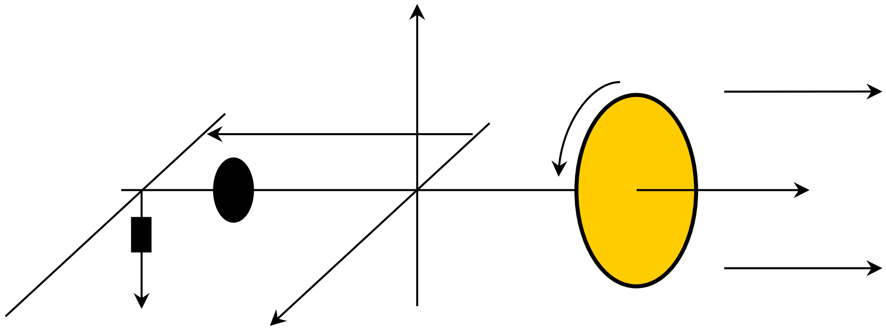


Figure 3. Schematic representation of the gyroscope submitted to an external torque

Due to the influence of the supplementary torque (which acts perpendicularly in this particular case), after a lapse of time *dt*, the angular momentum *L* will rotate by an angle $d\phi$ from its initial position (Figure 4).

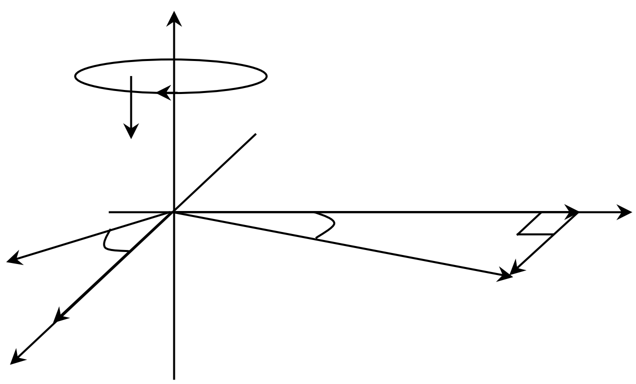


Figure 4. Precession of the horizontal axis of the gyroscope

$$\sin(d\phi) \approx d\phi = \frac{dL}{L} \tag{8}$$

The gyroscope does not topple under the influence of the supplementary torque, but reacts perpendicularly to the force generated by this torque. The gyroscope, which now is submitted to gravitation, describes the so-called precession movement. The angular velocity, ω_{pr}, of the precession fulfils the relation:

$$\omega_{pr} = \frac{d\phi}{dt} = \frac{1}{L}\frac{dL}{dt} = \frac{1}{I\omega_r}\frac{dL}{dt} = \frac{r^* m^* g}{I\omega_r} \qquad (9)$$

Taking $\omega_{pr} = 2\pi/T_{pr}$ and $\omega_r = 2\pi/T_r$ one obtains:

$$\frac{1}{T_r} = \frac{r^* m^* g}{4\pi^2}\frac{1}{I}T_{pr} \qquad (10)$$

where r* = 27cm

The double value of the torque (double value of *m**) causes the doubling of the precession frequency. If *m** is hung into the forward groove of the gyroscope axis, or if the direction of rotation of the disk is inverted, the direction of rotation of the precession is also inverted.

If the supplementary disk identical to the gyroscope disk is used too, and both are caused to rotate in opposite directions, no precession will occur when a torque is applied.

2-3- Determination of the nutation frequency

The relation between the nutation frequency ω_n and rotation frequency ω_r is given by:

$$\omega_n = k\omega_r \quad or \quad T_r = kT_n \qquad (11)$$

The constant k depends on the different moments of inertia relative to the principal axes of rotation.

3- List of equipment

Apparatus	Quantity	Catalogue Number
Gyroscope with 3 axes	1	02555.00
Light barrier with Counter	1	11207.30
Power supply 5 V DC/2.4 A	1	11076.99
Additional gyro-disc w. counter-weight	1	02556.00
Stopwatch, digital, 1/100 sec	1	03071.01
Barrel base -PASS-	1	02006.55
Slotted weight, 10 g, black	4	02205.01
Slotted weight, 50 g, black	1	02206.01
Holder for slotted weights	1	02204.00

4- Setup and carrying out the experiment

Part 1: determination of the momentum of inertia of the gyroscope disk

1- Set up the gyroscope as shown in Fig. 1 and balance it (be sure that the angle is 90°).

2- Suspend an additional weight from the axle with the help of a string (choose m = 20g)

3- Measure the height, h, of the mass from the ground, then leave it fall freely with computing the falling time, t_F, by using the stop watch.

4- Repeat step 3 for 6 different heights and fill Table 1.

Table 1. Determination of the moment of inertia from the slope $t_F^2 = f(h)$.

Height	*Falling time t_{Fi} (s)*			*Falling time average*	-------------
h (m)	*t_{F1} (s)*	*t_{F2} (s)*	*t_{F3} (s)*	$t_F = (t_{F1} + t_{F2} + t_{F3})/3$	t_F^2 *(s²)*
0.40					
0.50					
0.60					
0.70					
0.80					
0.90					

- Fill Table 2

Table 2. Determination of moment of inertia from the slope $t_F^2 = f(h)$ by using least squares method

h (m)	t_F^2 (s^2)	h^2 (m^2)	$(t_F^2)h$ ($m.s^2$)
0.40			
0.50			
0.60			
0.70			
0.80			
0.90			

$\overline{h} =$	$\overline{t_F^2} =$	$\overline{h^2} =$	$\overline{(t_F^2)h} =$

- Prepare a sheet of graph paper for plotting time squared t_F^2 versus distance *h*. You should make t_F^2 the vertical axis and distance *h* the horizontal axis. Each axis should be labeled and appropriate units indicated. The graph should have a title.

- Plot your data on the graph.

- Determine the slope, m*, and the y-intercept, b, of your best fit line.

- Draw best fit line to the points on your graph.

N.B: The best fit line must be drawn by using method of least squares (see appendix).

- Determine the moment of inertia, I, by using Equation (4) and the slope of your best fit line.

Part 2: determination of the precession frequency.

1- Set up the gyroscope as shown in Fig. 5 and balance it.

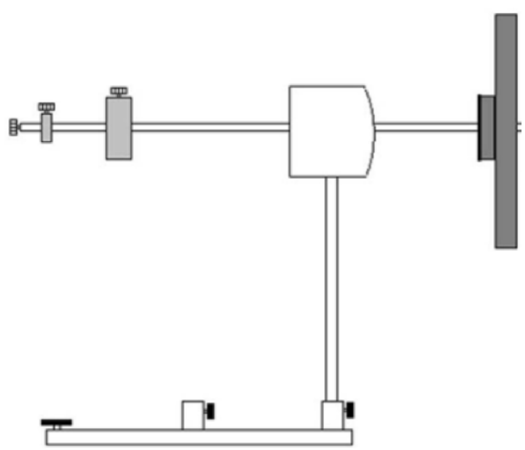


Figure 5. Precession

2- Make the device rotate either manually or with the help of the string.

3- Measure the duration of a revolution, T_r, of the gyroscope disk by using the light barrier and record it in table 3. Calculate the inverse of T_r.

4- Hang the mass (m* = 10g) on the end gyroscope axle. The gyroscope displays precession. Measure the time for half cycle by using stopwatch then from it determine the duration of a precession revolution T_p and record your result in table 3.

5- Repeat the steps from 2 to 4 for the different values of T_{pr}, in each time reduce the number of turns.

Table 3. Determination of the best fit line's slope and y-intercept of $T_r^{-1} = f(T_{pr})$ by using least squares method for **m* = 0.010 kg**

T_r (s)	T_{pr} (s)	$(T_r)^{-1}$ (s^{-1})	$(T_{pr})^2$ (s^2)	$(T_r^{-1})T_{pr}$
----------------	$\overline{T_{pr}}$ =	$\overline{T_r^{-1}}$ =	$\overline{T_{pr}^2}$ =	$\overline{(T_r^{-1})T_{pr}}$ =

- Prepare a sheet of graph paper for plotting the inverse of the duration of a revolution of the gyroscope disk, $(T_r)^{-1}$ versus the duration of a precession revolution time T_{pr}. You should make $(T_r)^{-1}$ the vertical axis and T_{pr} the horizontal axis. Each axis should be labeled and appropriate units indicated. The graph should have a title.

- Plot your data on the graph.

- Determine the slope, m*, and the y-intercept, b, of your best fit line.

- Draw the best fit line to the points on your graph.

N.B: The best fit line must be drawn by using method of least squares (see appendix).

- Determine the moment of inertia, I, by using Equation (10) and the slope of your best fit line.

- Repeat the above steps for mass m* = 0.020 kg and write your data in Table 4.

Table 4. Determination of the best fit line's slope and y-intercept of $T_r^{-1} = f(T_{pr})$ by using least squares method for **m* = 0.020 kg**

T_r (s)	T_{pr} (s)	$(T_r)^{-1}$ (s^{-1})	$(T_{pr})^2$ (s^2)	$(T_r^{-1})T_{pr}$
----------------	$\overline{T_{pr}} =$	$\overline{T_r^{-1}} =$	$\overline{T_{pr}^2} =$	$\overline{(T_r^{-1})T_{pr}} =$

- In the same sheet of graph paper as above, plot the inverse of the duration of a revolution of the gyroscope disk, $(T_r)^{-1}$ versus the duration of a precession revolution time T_{pr}.

- Plot your data on the graph.

- Determine the slope and the y-intercept of your best fit line.

- Draw the best fit line to the points on your graph.

N.B: The best fit line must be drawn by using method of least squares (see appendix).

- Determine the moment of inertia, I, by using Equation (10) and the slope of your best fit line.

Part 3: determination of the nutation frequency.

1. Set up the gyroscope as shown in Figure 5 and balance it.
2. Make the disc rotate either manually or with the help of the string.
3. A slight lateral push to the spinning axis of the gyroscope will initiate nutation.
4. In order to make a quantitative evaluation of the experiment, determine the period of a suitable number of nutation cycles.
5. Subsequently measure the period of rotation of the disc.
6. Make further measurements at slower disc frequencies by reducing the number of turns. Write your date in Table 5.

Table 5. Measure of T_n and T_r. Determination of the best fit line's slope and y-intercept of $T_r = f(T_n)$ by using least squares method.

T_n (s)	T_r (s)	$(T_n)^2$ (s^2)	$(T_n)(T_r)$ (s^2)

$\overline{T_n} =$	$\overline{T_r} =$	$\overline{T_n^2} =$	$\overline{(T_n)(T_r)} =$

- Prepare a sheet of graph paper for plotting the duration of a revolution of the gyroscope disk, T_r, versus the duration of a nutation revolution time T_n. You should make T_r the vertical axis and T_n the horizontal axis. Each axis should be labeled and appropriate units indicated. The graph should have a title.
- Plot your data on the graph.
- Determine the slope, m*, and the y-intercept, b, of your best fit line.
- Draw the best fit line to the points on your graph.

N.B: The best fit line must be drawn by using method of least squares (see appendix).

5- Conclusions

Discuss your results.