Experiment 2

Conservation of Energy by using Maxwell's Wheel

1- Objects of the experiment

- Introducing the concept of conservation of energy
- Measuring the transformation of potential energy into translational and rotational energy
- Determination of moment of inertia of the Maxwell's wheel

2- Principles

The total mechanical energy, E, of a system remains constant if the forces exerted on the system are conservative. This is known as "the law of conservation of mechanical energy". If the kinetic energy, K, of a conservative system increases (or decreases) by some amount, the potential energy, U, must decrease (or increase) by the same amount:

$$\Delta K + \Delta U = 0 \qquad (1)$$

Total mechanical energy of a system is the sum of the kinetic energy and potential energy of the system:

$$E = K + U \qquad (2)$$

which is constant during the motion. If the system is rolling, its total kinetic energy, K, is the sum of the translational kinetic energy, K_T, of its center of mass and the rotational kinetic energy K_R:

$$K = K_T + K_R \tag{3}$$

where the translational and the rotational kinetic energies are given as:

$$K_T = \frac{1}{2}mv^2 \tag{4}$$

$$K_R = \frac{1}{2}I\omega^2 \tag{5}$$

Here ω indicates the angular velocity, v translational velocity, and I the moment of inertia.

The potential energy is given by taking as reference U = 0 at s = 0:

$$U = mgs \tag{6-a}$$

or

$$U = -mg|s| \tag{6-b}$$

The Maxwell disk, which can unroll with its axis on two cords, moves in the gravitational field (Fig. 1).

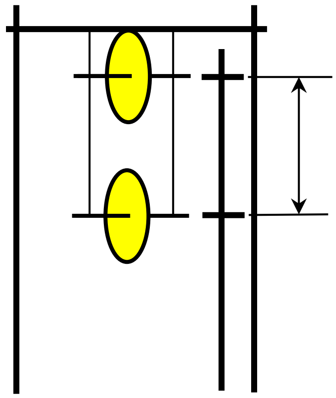

Figure 1. Experimental set up for investigating the conservation of energy, using the Maxwell's disk.

The total energy of the Maxwell's disk is composed of the potential energy, translational energy and rotational energy:

$$E = U + K_T + K_R = mgs + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad (6)$$

The relationship between translational and angular velocities (Fig. 2) is given as

$$ds = r\,d\theta \qquad (7)$$

and

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega \qquad (8)$$

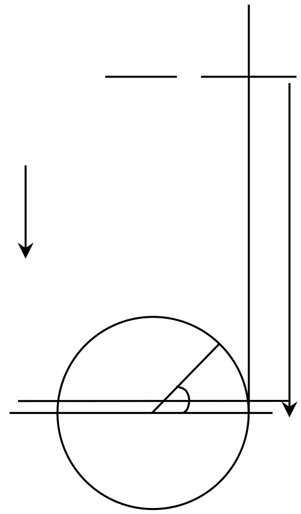

Figure 2. Relationship between the increase in angle and the decrease in height in the Maxwell disk.

Then total energy of the system becomes

$$E = mgs + \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = mg\,s(t) + \frac{1}{2}\left(m + \frac{I}{r^2}\right)v^2(t) \qquad (9)$$

Since the total energy of the system is constant over time, differentiation gives

$$\frac{dE}{dt} = 0 = mg\,v(t) + \left(m + \frac{I}{r^2}\right)v(t)\frac{dv(t)}{dt} \qquad (10)$$

For *s(t=0)=0* and *v(t=0)=0*, one obtains the velocity and the displacement as

$$v(t) = -\frac{mg}{\left(m + \frac{I}{r^2}\right)}t \qquad (11)$$

and

$$s(t) = -\frac{1}{2}\frac{mg}{\left(m + \frac{I}{r^2}\right)}t^2 \qquad (12)$$

The mass *m* of the Maxwell's wheel is $m = 0.436\ kg$, the radius *r* of the spindle is $r = 2.5\ mm$, and the gravitational acceleration is $g = 9.81\ m/s^2$.

3- List of equipment

Apparatus	Quantity	Catalogue Number
Support base –PASS-	1	02005.55
Support rod –PASS-, square, l=1m	2	02028.55
Right angle clamp –PASS-	2	02040.55
Meter scale, demo, l=1m	1	03001.00
Cursors, 1 pair	1	02201.00
Maxwell wheel	1	02425.00
Stopwatch, digital, 1/100 sec	1	03071.01

4- Setup and procedure

The experimental setup is as shown in Figure 1. Using the adjusting screw on the support rod, the axis of the Maxwell's wheel, in the unwound condition, is aligned horizontally, when winding up, the windings must run inwards.

The winding density should be approximately equal on both sides. It is essential to watch the first up and down movements of the disk, since incorrect winding (outwards, crossed over) will cause the "gyroscope" to break free.

a- Distance travelled by the centre of gravity of the Maxwell's wheel as a function of time.

- Measure the distance, $|s|$, that the Maxwell's wheel travel during measurement and record it on Table 1.

- Release the disk and using a counter (chronometer) measure the time required for that distance. Repeat each measurement 3 times. Record the measurements on table 1.

Table 1: Selected distance $|s|$, and measured time *t*.

$\|s\|$ (m)	t_1 (s)	t_2 (s)	t_3 (s)	$t=(t_1+t_2+t_3)/3$ (s)	t^2 (s^2)
0.20	………	………	………		
0.25	………	………	………		
0.30	………	………	………		
0.35	………	………	………		
0.40	………	………	………		
0.45	………	………	………		
0.50	………	………	………		
0.55	………	………	………		

- Fill Table 2

Table 2: Determination of the best fit line's slope and y-intercept of $|s| = f(t^2)$ by using least squares method

$\|s\|$	t^2	$(t^2)^2$	$(t^2)\|s\|$

(m)	(s^2)	(s^4)	($m.s^2$)
0.20			
0.25			
0.30			
0.35			
0.40			
0.45			
0.50			
0.55			
$\overline{\lvert s\rvert} =$	$\overline{t^2} =$	$\overline{(t^2)^2} =$	$\overline{(t^2)\lvert s\rvert} =$

- Prepare a sheet of graph paper for plotting **distance** $\lvert s\rvert$ versus time squared t^2. You should make $\lvert s\rvert$ the vertical axis and t^2 the horizontal axis. Each axis should be labeled and appropriate units indicated. The graph should have a title.

- Plot your data on the graph.

- Determine the slope, m*, and the y-intercept, b, of your line.

- Draw best fit line to the points on your graph.

N.B: The best fit line must be drawn by using method of least squares (see appendix).

- Determine the moment of inertia, I, by using Equation 12 and the slope of your best fit line.

b- Total energy of the Maxwell disk as a function of time.

- Calculate the linear speed of the wheel from the Eq. (11). Record the values on the table 3.
- Find the potential energy for each time using the Eq. (6-b) and record it on the table 3.
- Calculate the translational kinetic energy for each time using the Eq. (4) and record it on the table 3.
- Calculate the rotational kinetic energy for each time using the Eqs. (5 and 8) and record it on the table 3.
- Now calculate the mechanical energy of whole system for each time using the Eq. (2) and record it on the table 3.

Table 3: Calculation of speed and energies

$\|s\|$ (m)	t (s)	$\|v\|$ (m/s) *Eq. 11*	U (J)	K_T (J)	K_R (J)	E (J)
0.20						
0.25						
0.30						
0.35						
0.40						
0.45						
0.50						
0.55						

- Fill Table 4.

Table 4. Determination of the best fit line's slope and y-intercept of $E = f(t)$ by using least squares method.

$\lvert s \rvert$ (m)	t (s)	E (J)	t^2	Et
0.20				
0.25				
0.30				
0.35				
0.40				
0.45				
0.50				
0.55				
	$\bar{t} =$	$\overline{E} =$	$\overline{t^2} =$	$\overline{Et} =$

- Prepare a sheet of graph paper for plotting total energy, E, versus time t. You should make E the vertical axis and t the horizontal axis. Each axis should be labeled and appropriate units indicated. The graph should have a title.

- Plot your data on the graph.

- Determine the slope, m*, and the y-intercept, b, of your line.

- Draw best fit line to the points on your graph.

N.B: The best fit line must be drawn by using method of least squares (see appendix).

5) Conclusions

- Discuss your results.