Motion in one Dimension

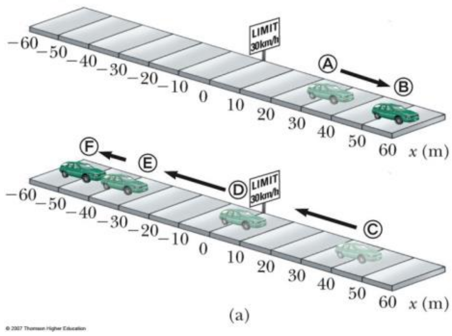
Chapter 2 : Motion in1 D

Chapter Outline:

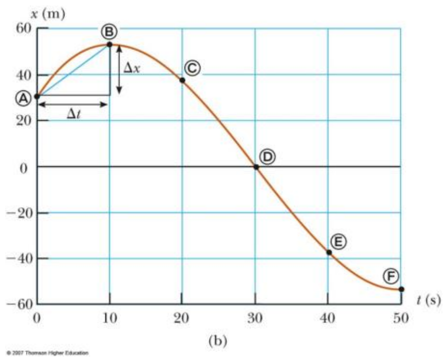
1- Position and Displacement
2- Velocity
3- Acceleration
4- Motion of objects traveling with Constant Acceleration
5- Free Falling Objects

2.1 Position, Velocity, and Speed:

A particle's **position** x is the location of the particle with respect to a chosen reference point that we can consider to be the origin.



(a)



(b)

2.1 Position, Velocity, and Speed:

- The table gives the actual data collected during the motion of the object (car)
- Positive is defined as being to the right

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

2.1 Position, Velocity, and Speed:

The **displacement** Δx of a particle is defined as its change in position in some time interval. As the particle moves from an initial position x_i to a final position x_f , its displacement is given by

$$\Delta x \equiv x_f - x_i \qquad (1)$$

- We use the capital Greek letter delta (Δ) to denote the *change* in a quantity.
- Δx is positive if x_f is greater than x_i.
- Δx is negative if x_f is less than x_i.
- SI units are meters (m)
- Displacement is an example of a vector quantity.

- we use positive(+) and negative (-) signs to indicate vector direction. For example, for horizontal motion the right is the positive direction. It follows that any object always moving to the right undergoes a positive displacement $\Delta x>0$
- Any object moving to the left undergoes a negative displacement so that $\Delta x<0$.

2.1 Position, Velocity, and Speed:

The **average velocity** $v_{x,avg}$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} \qquad (2)$$

The **average speed** v_{avg} of a particle, a scalar quantity, is defined as the total distance d traveled divided by the total time interval required to travel that distance:

$$v_{avg} \equiv \frac{d}{\Delta t} \qquad (3)$$

The SI unit of average speed is the same as the unit of average velocity: meters per second.

Unlike average velocity, however, average speed has no direction and is always expressed as a positive number.

Example

Example1
A particle at t1 = -2.0 s is at x1= 3.4 cm and at t2 = 4.5 s is at x2= 8.5 cm. What is its average velocity? Can you calculate its average speed from these data?

Example 2
An airplane travels 3100 km at a speed of 790 km/h and then encounters a tailwind that boosts its speed to 990 km/h for the next 2800 km. What was the total time for the trip?
What was the average speed of the plane for this trip?

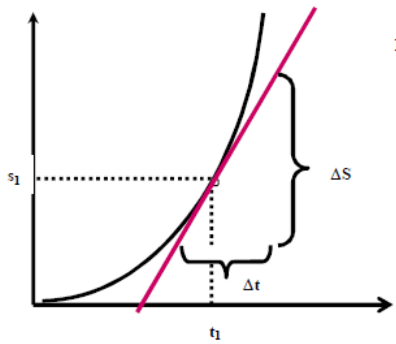
2.2 Instantaneous Velocity and Speed:

* The instantaneous velocity is the velocity of an object at some instant or at a specific point in the object's path.
* The instantaneous velocity at a given time can be determined by measuring the slope of the line that is tangent to that point on the position-versus-time graph.

The **instantaneous velocity** v_x is :

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (4)$$

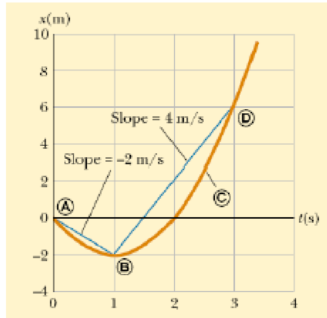
The instantaneous velocity can be positive, negative, or zero. The **instantaneous speed** of a particle is defined as the magnitude of its instantaneous velocity.



2.2 Instantaneous Velocity and Speed:

A particle moves along the x axis. Its x coordinate varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds. The position–time graph for this motion is shown in the Figure. Note that the particle moves in the negative x direction for the first second of motion, is at rest at the moment $t = 1$ s, and moves in the positive x direction for $t > 1$ s.

(a) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.
(b) Calculate the average velocity during these two time intervals.
(c) Find the instantaneous velocity of the particle at $t = 2.5$ s.



Particle Under Constant Velocity

If the velocity of a particle is constant, its instantaneous velocity at any instant during a time interval is the same as the average velocity over the interval. That is, $v_x = v_{x,avg}$.

Therefore, Equation (2) gives us an equation to be used in the mathematical representation of this situation:

$$v_x = \frac{\Delta x}{\Delta t} \qquad (6)$$

Remembering that $\Delta x = x_f - x_i$, we see that $v_x = (x_f - x_i)/\Delta t$, or

$x_f = x_i + v_x \Delta t$

In practice, we usually choose the time at the beginning of the interval to be $t_i = 0$ and the time at the end of the interval to be $t_f = t$, so our equation becomes

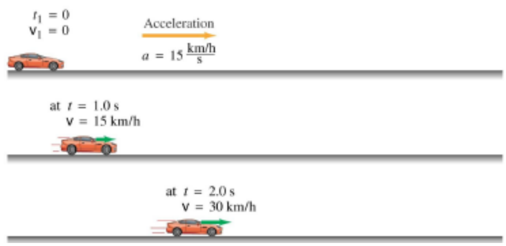
$$x_f = x_i + v_x t \qquad (7)$$

2.3 Acceleration:

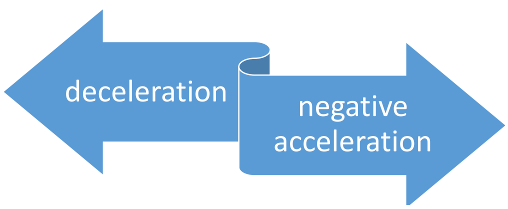
The **average acceleration** ***of the particle*** is defined as the *change in velocity* Δv_x *divided by the time interval* Δt *during which* that change occurs:

$$\boxed{a_{x,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}}$$

As with velocity, when the motion being analyzed is one dimensional, we can use positive and negative signs to indicate the direction of the acceleration.



2.3 Acceleration:



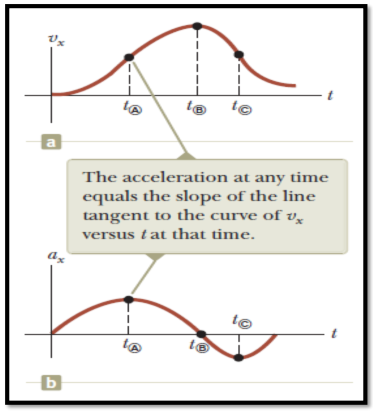
	Speeding up (+)	Slowing down (-)
+ velocity	+a	-a
- velocity	-a	+a

When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.

2.3 Acceleration:

the **instantaneous acceleration** as the limit of the average acceleration as Δt *approaches zero:*

$$\boxed{a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}}$$



2.3 Acceleration:

Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

(B) Determine the acceleration at $t = 2.0$ s.

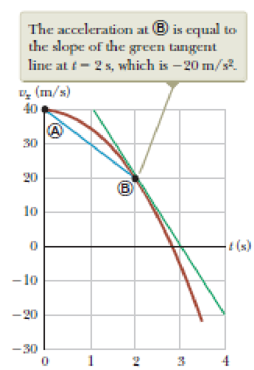
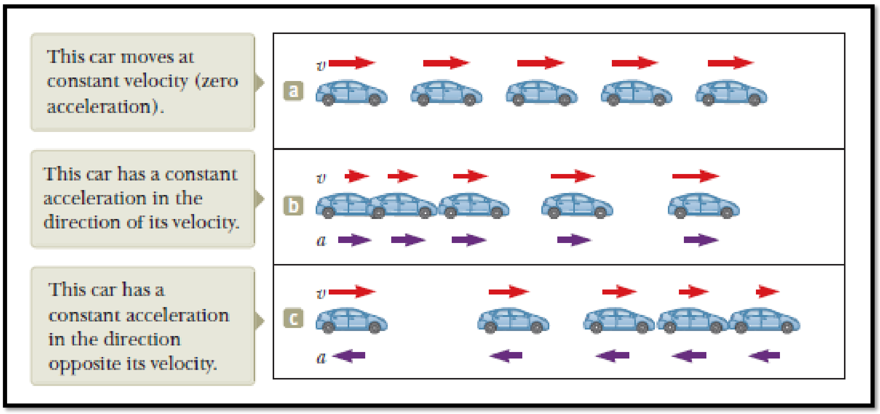


Figure 2.9 (Example 2.6) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = 40 - 5t^2$.

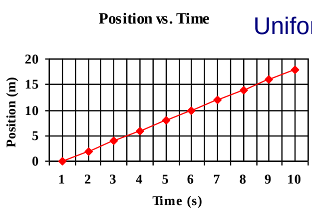
2.4 Motion Diagrams:

motion diagram is sometimes useful to describe the velocity and acceleration while an object is in motion.



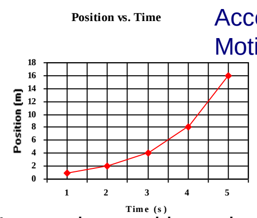
Position. Vs. Time graphs

Uniform Motion

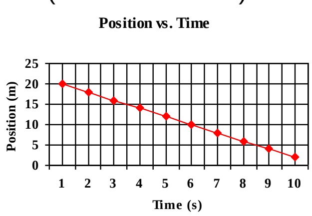


Constant positive velocity
(zero acceleration)

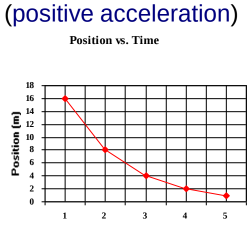
Accelerated Motion



Increasing positive velocity
(positive acceleration)

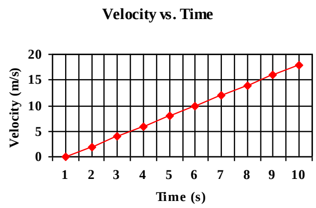


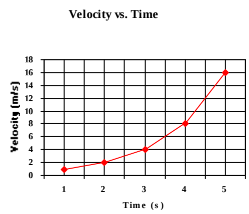
Constant negative velocity
(zero acceleration)

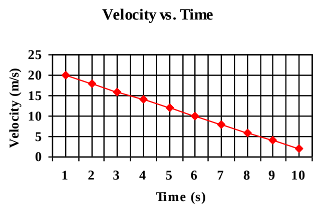


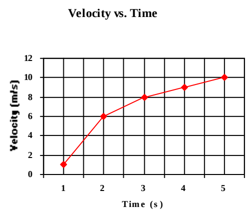
Decreasing negative velocity
(positive acceleration)

Velocity-time graphs

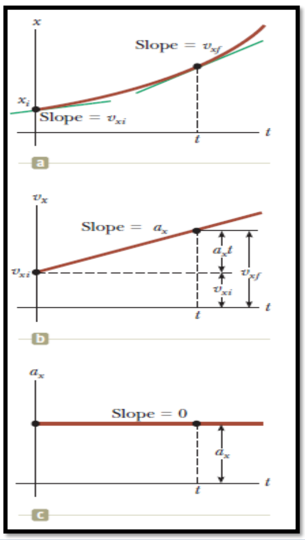








2.6 1D Montion Under Constant Acceleration:



2.6 1D Montion Under Constant Acceleration:

When acceleration is constant. The average acceleration $a_{x,avg}$ over any time interval is equal to the instantaneous acceleration a_x at any instant within the interval.

$$a_x = \frac{\Delta v}{\Delta t}$$

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

or

$$v_{xf} = v_{xi} + a_x t \quad (for\ constsnt\ a_x)\ (1)$$

2.6 1D Montion Under Constant Acceleration:

In addition, as the velocity is increasing at a constant rate, we know that

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2} \ (for\ constsnt\ a_x)$$

Notice that this expression for average velocity applies *only* if the acceleration is constant.

To obtain the position of an object as a function of time, Recalling that Δx in $v_{x,avg} \equiv \frac{\Delta x}{\Delta t}$ represents $xf - xi$ and recognizing that $\Delta t = tf - ti = t - 0 = t$, we find that:

$$x_f - x_i = v_{x,avg} t = \frac{1}{2}(v_{xi} + v_{xf})t$$

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t \ \ (for\ constant\ a_x) \qquad (2)$$

2.6 1D Montion Under Constant Acceleration:

We can obtain another useful expression for the position of a particle under constant acceleration by substituting Equation (1) into Equation (2):

$$x_f = x_i + \frac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad (for\ constant\ a_x) \quad (3)$$

Finally, we can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from Equation (1) into Equation (2):

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right)$$

$$v_f^2 = v_i^2 + 2a_x(x_f - x_i) \quad (for\ constant\ a_x) \quad (4)$$

2.6 1D Montion Under Constant Acceleration:

For motion at *zero* acceleration, we see from Equations (1) and (3) that:

$$\left.\begin{aligned} v_{xf} = v_{xi} = v_x \\ x_f = x_i + v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when the acceleration of a particle is zero, its velocity is constant and its position changes linearly with time.

TABLE 2.2 *Kinematic Equations for Motion of a Particle Under Constant Acceleration*

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

2.6 1D Montion Under Constant Acceleration:

Example 2.7 Carrier Landing

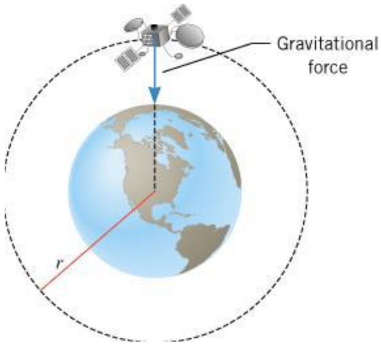
A jet lands on an aircraft carrier at a speed of 140 mi/h ($\approx$ 63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

(B) If the jet touches down at position $x_i = 0$, what is its final position?

2.7 Free Fall

The acceleration of a freely falling body is called the ***acceleration due to gravity,*** *g*. The acceleration due to gravity is directed downward, toward the center of the earth. Near the earth's surface, $g = 9.80$ m/s2.

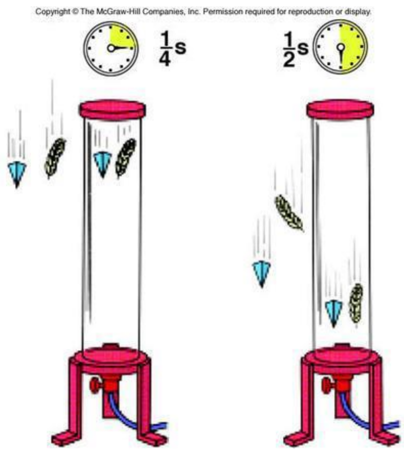


2.7 Free Fall

Heavy and light objects fall at the same rate

- **In air…**
  – A stone falls faster than a paper
  - Air resistance affects stone less

- **In a vacuum**
  – A stone and paper will fall at the same speed



2.7 Free Fall

Any freely falling object experiences an acceleration directed downward, regardless of its initial motion. We shall take the vertical direction to be the *y axis* and called positive *y* upward.

$$a_y = -g = -9.80\ m/s^2,$$

$$v = v_0 - g\,t$$

$$y - y_0 = \frac{1}{2}(v + v_0)t$$

$$y - y_0 = v_0 t - \frac{1}{2} g\,t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Example 2.10 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in Figure 2.14.

(A) Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine the time at which the stone reaches its maximum height.

(B) Find the maximum height of the stone.

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

(D) Find the velocity and position of the stone at $t = 5.00$ s.