Chapter 5
Current and Resistance

1- Electric Current

The **electric current** I in a conductor is defined as

$$I \equiv \frac{dQ}{dt}$$

where dQ is the charge that passes through a cross-section of the conductor in a time dt.

The SI unit of current is the **ampere** (A), where 1A=1C/s.

2- Sign Convention for Electric Current

The direction of current flow is opposite to the direction of electron flow.

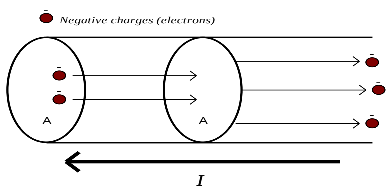
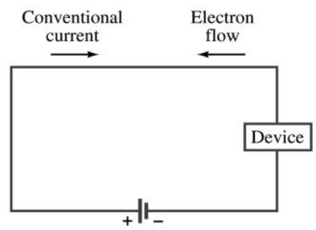
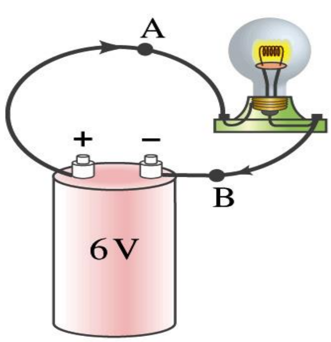


Figure. A wire of cross-sectional area A

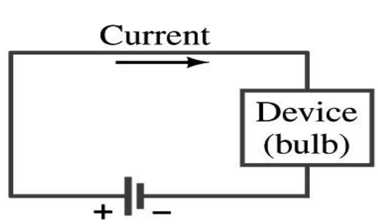


2- Sign Convention for Electric Current

Positive current flows from the positive terminal through the conductors and device (load) back to the negative terminal.



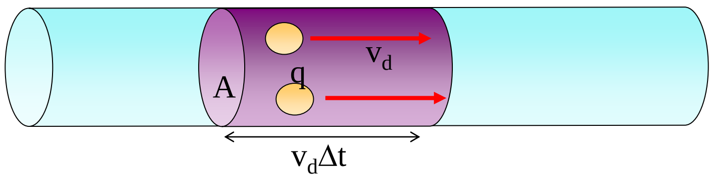
(a)



(b)

3- Current and Drift Speed

Consider the current on a conductor of cross-sectional area A.



- Volume of an element of length Δx is : $\Delta V = A\, \Delta x$.
- Let n be the number of carriers per unit of volume.
- The total number of carriers in ΔV is: $n\, A\, \Delta x$.
- The charge in this volume is: $\Delta Q = (n\, A\, \Delta x)q$.
- Distance traveled at drift speed v_d by carrier in time Δt:

$$\Delta x = v_d\, \Delta t.$$

- Hence: $\Delta Q = (n\, A\, v_d\, \Delta t)q$.
- The current through the conductor:

$$I = \Delta Q / \Delta t = n\, A\, v_d\, q$$

3- Current and Drift Speed

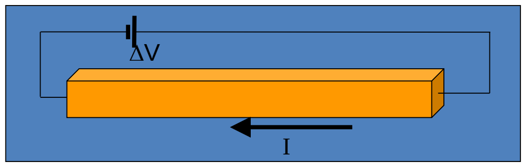
Question:

A copper wire of cross-sectional area 3.00×10^{-6} m^2 carries a current of 10. A. Assuming that each copper atom contributes one free electron to the metal, find the drift speed of the electron in this wire. The density of copper is 8.95 g/cm^3.

4 Resistance and Ohm's Law

When a voltage (potential difference) is applied across the ends of a metallic conductor, the current is found to be proportional to the applied voltage.

$$I \propto \Delta V$$



In situations where the proportionality is exact, one can write.

$$\Delta V = IR$$

•The proportionality constant R is called resistance of the conductor.
•The resistance is defined as the ratio.

$$R = \frac{\Delta V}{I}$$

4 Resistance

In SI, resistance is expressed in volts per ampere.
A special name is given: ohms (Ω).

Example:
If a potential difference of 10 V applied across a conductor produces a 0.2 A current, the conductors resistance is?

4 Resistance

Consider a conductor of cross-sectional area *A* carrying a current *I*. The current density *J* in the conductor is defined as the current per unit area.

Because the current $I = nqv_d A$,

the current density is $J \equiv \frac{I}{A} = nqv_d$

where *J* has SI units of A/m2. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area *A* is perpendicular to the direction of the current. In general, current density is a vector quantity:

$$\vec{J} = nq\vec{v}_d$$

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor.

4 Resistance

In some materials, the current density is proportional to the electric field:

$$\vec{J} = \sigma\vec{E}$$

where the constant of proportionality σ is called the conductivity of the conductor.

A potential difference $\Delta V = Vb - Va$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the potential difference is related to the field through the relationship

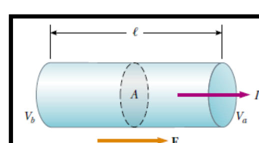
$$\Delta V = El$$

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{l}$$

Because $J = I/A$, we can write the potential difference as

$$\Delta V = \frac{l}{\sigma} J = \left(\frac{l}{\sigma A}\right) I = RI$$



The inverse of conductivity is resistivity ρ :

$$\rho = \frac{1}{\sigma}$$

4 -Resistance

The resistance of an ohmic conductor is proportional to the its length, *l*, and inversely proportional to the cross section area, *A*, of the conductor.

$$R = \rho \frac{l}{A}$$

The constant of proportionality ρ is called the resistivity of the material.
المقاومة النوعية لمادة: Resistivity

Resistivity – Units

$$R = \rho \frac{l}{A} \Rightarrow \rho = \frac{RA}{l}$$

Resistance expressed in Ohms,
Length in meter.
Area are m2,
Resistivity thus has units of Ωm.

5- Resistivity

Material	Resistivity (10^{-8} Ωm)	Material	Resistivity (10^{-8} Ωm)
Silver	1.61	Bismuth	106.8
Copper	1.70	Plutonium	141.4
Gold	2.20	Graphite	1375
Aluminum	2.65	Germanium	$4.6x10^{7}$
Pure Silicon	3.5	Diamond	$2.7x10^{9}$
Calcium	3.91	Deionized water	$1.8x10^{13}$
Sodium	4.75	Iodine	$1.3x10^{15}$
Tungsten	5.3	Phosphorus	$1x10^{17}$
Brass	7.0	Quartz	$1x10^{21}$
Uranium	30.0	Alumina	$1x10^{22}$
Mercury	98.4	Sulfur	$2x10^{23}$

5- Resistivity - Example

Example 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of 2.00×10^{-4} m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot$m.

Example 27.3 The Resistance of Nichrome Wire

(A) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Power

Compute rate of energy loss (power dissipated on the resistor)

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta V = I\Delta V$$

Use Ohm's law

$$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

Units of power: SI: watt

Power

A high-voltage transmission line with resistance of 0.31 Ω /km carries 1000A , starting at 700 kV, for a distance of 160 km. What is the power loss due to resistance in the wire?

Resistors in Series

- The current is the same in all resistors because any charge that flows through R_1 flows through R_2 .

$$I = I_1 = I_2$$

- The sum of the potential differences across the resistors is equal to the total potential difference across the combination.

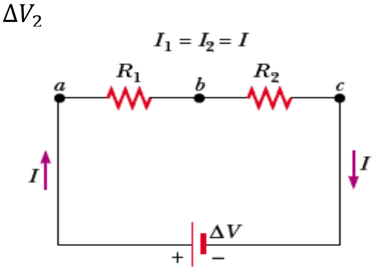
$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = IR = I_1R_1 + I_2R_2 = I(R_1 + R_2)$$

$$R_{eq} = R_1 + R_2$$

If there are more than two resistors:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$



The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any of the individual resistors.

Resistors in Parallel

- A junction is any point in a circuit where a current can split.

$$I = I_1 + I_2$$

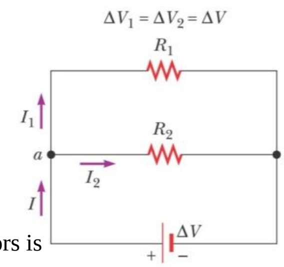
- The potential difference across each resistor is the same because each is connected directly across the battery terminals.

$$\Delta V = \Delta V_1 = \Delta V_2$$

$$I = \frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$



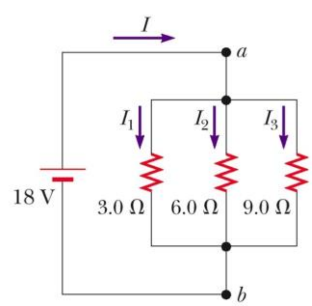
The equivalent resistance of a parallel combination of resistors is always less than any of the individual resistors

Resistors

a. Find the equivalent resistor.

b. Find the current in each resistor ,total current (I)

c. Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.



Resistors

partA:

$$\frac{1}{R_{eq}}=\frac{1}{3\Omega}+\frac{1}{6\Omega}+\frac{1}{9\Omega}=\frac{6+3+2}{18}=\frac{11}{18}$$

$$R_{eq}=\frac{18}{11}\ \Omega$$

PartB:

$$I_1=\frac{V}{R_1}=\frac{18\ V}{3\ \Omega}=6\ A$$

$$I_2=\frac{V}{R_2}=\frac{18\ V}{6\ \Omega}=3\ A$$

$$I_3=\frac{V}{R_3}=\frac{18\ V}{9\ \Omega}=2\ A$$

$$I=I_{total}=\frac{V}{R_{eq}}=\frac{18\ V}{\frac{18}{11}\ \Omega}=11\ A$$

$$I=I_1+I_2+I_3$$

$$I=6+3+2=11\Omega$$

PartC:

$$P_1=I_1^2R_1=(6A)^2(3\Omega)=108W$$

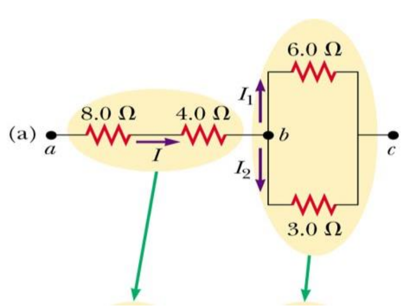
$$P_2=I_2^2R_2=(3A)^2(6\Omega)=54W$$

$$P_3=I_3^2R=(3A)^2(9\Omega)=36W$$

Resistors

A. Find the equivalent resistance between points a and c.

B. What is the current in each resistor if a potential difference of 42V is maintained between a and c ?



Resistors

$$I = \frac{\Delta V}{R_{eq}} = \frac{42}{14} = 3\ A$$