Chapter 24
Electric potential

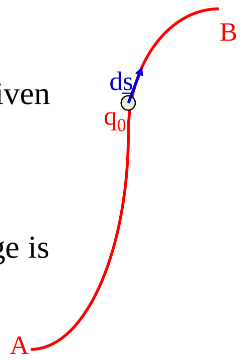
25.1 Potential Difference and Electric Potential

i) Change in Potential Energy

Consider a test charge "q_0" placed in an electric field "E".

The electric force acting on the test charge is given by:

$$\vec{F} = q_o \vec{E}$$



The work done by the electric field on the charge is given by:

$$dW = \vec{F} \cdot d\vec{s} = q_o \vec{E} \cdot d\vec{s}$$

25.1 Potential Difference and Electric Potential

This decreases the potential energy of the field-charge system by a quantity dU, where

$$dU = -dW \qquad dU = -q_o \vec{E} \cdot d\vec{s}$$

For a finite displacement of the test charge q_0 from a point A to a point B, the change in the potential energy, $\Delta U = UB - UA$, is given by:

$$\Delta U = \int_A^B dU = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

The integration is performed along the path by which q_0 moves from A to B and is called path integral or line integral.

25.1 Potential Difference and Electric Potential

ii) Potential Difference

The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a value at every point in an electric field, is called the electric potential (or simply the potential) V.

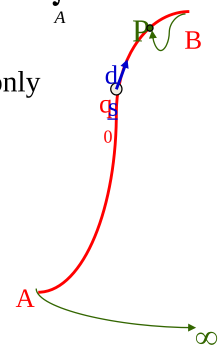
$$\Delta V = \frac{\Delta U}{q_0}$$

The Potential energy is a scalar quantity, and The electric potential also is a scalar quantity.

25.1 Potential Difference and Electric Potential

The potential difference between "B" and "A" $\Delta V = V_B - V_A$ defined as the change in potential energy divided by the test charge q_0. and given by :

$$\Delta V = \frac{\Delta U}{q_0} = \frac{-q_0 \int_A^B \vec{E} \cdot d\vec{s}}{q_0} \Rightarrow \Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$



- The potential difference between A and B depends only on the source charge
- The potential at infinity is considered to be zero
- The potential difference between a point "P" and
- "∞" is given by :

$$V_P - V_\infty = -\int_\infty^P \vec{E} \cdot d\vec{s} \Rightarrow V_P = -\int_\infty^P \vec{E} \cdot d\vec{s}$$

The quantity V_P is called the potential of point P.

25.1 Potential Difference and Electric Potential

Units

The SI unit of electric potential is joules

The SI unit of potential difference is joules per coulomb,

which is defined as a volt (V):

$$[V] = \frac{J}{C}$$

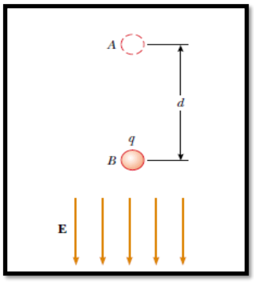
That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Example 1:

An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37 x 10^{-17} J. Calculate the charge on the ion.

25.2 Potential Differences in a Uniform Electric Field

Consider a uniform electric field directed along the negative y axis, as shown in Figure a. Let us calculate the potential difference between two points A and B separated by a distance , magnitude |s| = *d* , where s is parallel to the field lines.



$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B (E \cos 0°)\, ds = -\int_A^B E\, ds$$

25.2 Potential Differences in a Uniform Electric Field

- Because E is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E\int_A^B ds = -Ed$$

- The negative sign indicates that the electric potential at point B is lower than at point A; that is, *VB* < *VA*.
- Electric field lines always point in the direction of decreasing electric potential.

25.2 Potential Differences in a Uniform Electric Field

- Now suppose that a test charge q0 moves from A to B. We can calculate the change in the potential energy of the charge–field system:

$$\Delta U = q_0\,\Delta V = -q_0 Ed$$

- From this result, we see that if

$$q_0 : Positive \Rightarrow \Delta U : Negative$$

Electric field does the work on $(+q_0)$ when it moves in its direction.

The $(+q_0)$ gains kinetic energy and accelerates in the direction of "E".

$$q_0 : Negative \Rightarrow \Delta U : Positive$$

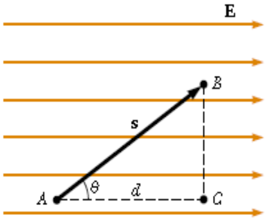
$(-q_0)$ gains potential energy when it moves in the direction of electric field.

External agency do the work to move the $(-q_0)$ in the direction of "E".

25.2 Potential Differences in a Uniform Electric Field

Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector s is not parallel to the field lines, as shown in the Figure below.

A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the same electric potential.



Then

$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{s}$$

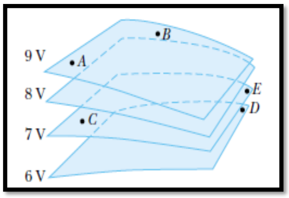
The change in potential energy of the charge–field system is

$$\Delta U = q_0\,\Delta V = -q_0\,\mathbf{E}\cdot\mathbf{s}$$

25.2 Potential Differences in a Uniform Electric Field

We conclude that all points in a plane perpendicular to a uniform electric field are at the same electric potential.

The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.



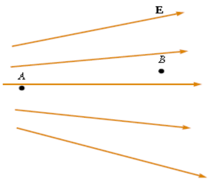
Quiz

2: In the Figure, a negative charge is placed at A and then moved to B. The change in potential energy of the charge–field system for this process is

(a) positive,

(b) negative,

(c) zero.



1: In the Figure, two points A and B are located within a region in which there is an electric field. The potential difference $\Delta V = V_B - V_A$ is:

(a) positive

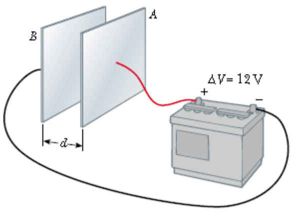
(b) negative

(c) zero

Example 2 :

A 12 V battery is connected between two parallel plates. The separation between the plates is d= 0.3 cm.

Find the magnitude of the electric field between the plates (assuming that E is uniform) ?



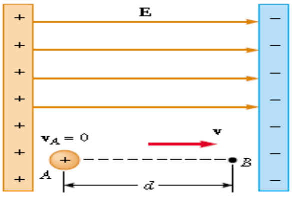
Example 3:Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of 8.0 x10^4 V/m . The proton undergoes a displacement of 0.50 m in the direction of E.

(A) Find the change in electric potential between points A and B.

(B) Find the change in potential energy of the proton–field system for this displacement.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.



25.3 Electric Potential and Potential Energy Due to Point Charges

To find the electric potential at a point located a distance r from the charge, we begin with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

where A and B are the two arbitrary points. At any point in space, the electric field due to the point charge is

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

where r̂ is a unit vector directed from the charge toward the point.

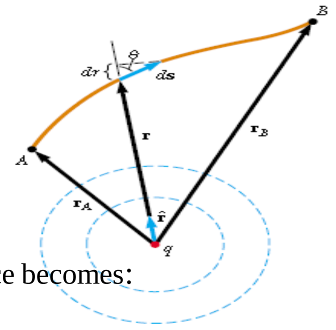
The quantity E . ds can be expressed as

$$\vec{E} \cdot d\vec{s} = k_e \frac{q}{r^2} \hat{r} \cdot d\vec{s}$$

25.3 Electric Potential and Potential Energy Due to Point Charges

Note

$$\hat{r}\cdot d\vec{s} = |\hat{r}||d\vec{s}|\cos\theta$$
$$\hat{r}\cdot d\vec{s} = ds\cos\theta = dr$$



Making these substitutions, we find that

$$\vec{E}\cdot d\vec{s} = k_e\frac{q}{r^2}dr$$

And the expression for the potential difference becomes:

$$V_B - V_A = -k_e q\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{k_e q}{r}\Big]_{r_A}^{r_B}$$
$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

25.3 Electric Potential and Potential Energy Due to Point Charges

If point *A* is located at infinity, then

$$r_A = \infty, \quad \frac{1}{r_A} = 0, \quad V_A = 0$$

$$V_B = k_e\frac{q}{r_B}$$

The electric potential *V* at a point distant *r* from a point charge *q* is given by :

$$V = k_e\frac{q}{r}$$

25.3 Electric Potential and Potential Energy Due to Point Charges

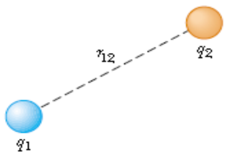
For a group of point charges, we can write the total electric potential at *P* in the form

$$V = k_e \sum_i \frac{q_i}{r_i}$$

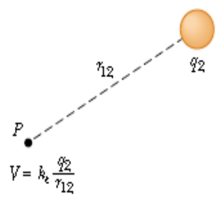
where the potential is again taken to be zero at infinity and r_i is the distance from the point P to the charge q_i .

25.3 Electric Potential and Potential Energy Due to Point Charges

We now consider the potential energy of a system of two charged particles. If *V*2 is the electric potential at a point *P* due to charge *q*2, then the work an external agent must do to bring a second charge *q*1 from infinity to *P* without acceleration is *q*1*V*2.
Therefore, we can express the potential energy of the system as



$$U = k_e \frac{q_1 q_2}{r_{12}}$$



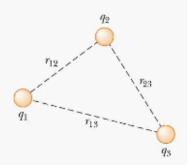
Note that if the charges are of the same sign, U is positive. This is consistent with the fact that positive work must be done by an external agent on the system to bring the two charges near one another (because charges of the same sign repel).

If the charges are of opposite sign, U is negative; this means that negative work is done by an external agent against the attractive force between the charges of opposite sign as they are brought near each other—a force must be applied opposite to the displacement to prevent q_1 from accelerating toward q_2 .

25.3 Electric Potential and Potential Energy Due to Point Charges

The total potential energy of the system of three charges

$$U = k_e \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$



The total potential energy of the system of four charges

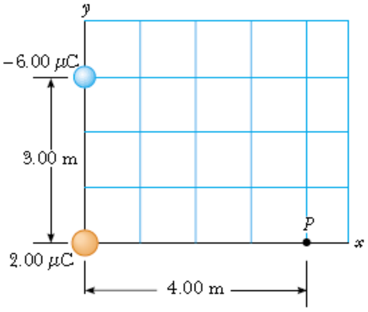
$$U = k_e \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}} \right]$$

Example 4 :

A charge q_1 = 2.00 µC is located at the origin, and a charge q_2 = − 6.00 µC is located at (0, 3.00) m, as shown in Figure a.

(A)Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

(B) Find the change in potential energy of the system of two charges plus a charge q_3 =3.00 µC as the latter charge moves from infinity to point P (Fig. b).



(a)