Chapter 2 :
Faraday`s law

Chapter 2 : **Faraday`s law**

Chapter 2 Outline:

Ch. 2 Faraday`s law
(Ch. 31 of the text book)

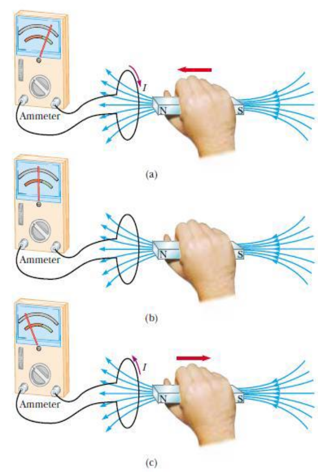
2.1 Faraday’s Law
2.2 Some Applications of Faraday’s Law
2.3 Motional emf
2.4 Lenz’s Law
2.5 Induced emf, Electric Fields and Applications
2.6 Maxwell’s Equations

2.1 Faraday's Law

when a north or south pole of magnet moved perpendicular to a closed conductor loop, a current is generated and its direction depends on the magnetic field direction.

Note. Static magnetic field can not generate the current

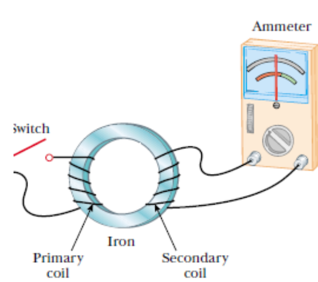
The results are quite remarkable in view of the fact that a current is set up even though no batteries are present in the circuit! We call such a current an induced current and say that it is produced by an induced emf.



2.1 Faraday's Law

Another way to change the magnetic field across closed loop is to produce a magnetic field by a coil.
Nearby the coil the secondary coil in which any change in magnetic field caused by the primary coil generates a current in the secondary coil as shown in the simple transformer

Note: constant current in the primary coil will not produce a current in the secondary coil



2.1 Faraday's Law

In a closed loop conductor, the induced electromotive force (ε) is directly proportional to the negative rate of change in magnetic flux

$$\varepsilon = -\frac{d\phi_B}{dt}$$

For several loops connected coil or number of turn is N, the emf is then the multiplication of N by the negative rate change of magnetic flux

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

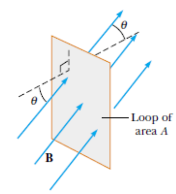
If the magnetic field is not perpendicular to the coil (sloped with angle θ), the magnetic flux is then determined by :-

$$\phi_B = \int BdA = BA\cos\theta$$

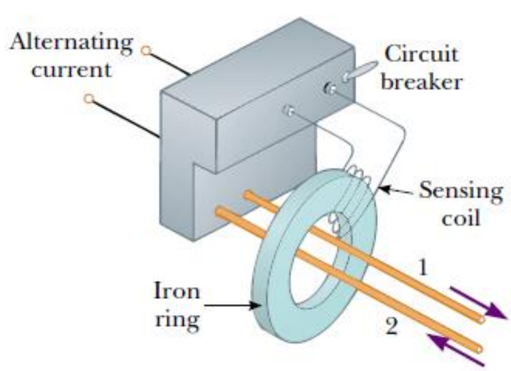
$$and \quad \varepsilon = -\frac{d}{dt}(BA\cos\theta)$$

we see that an emf can be induced in the circuit in several ways

- The magnitude of B can change with time.
- The area enclosed by the loop can change with time.
- The angle θ between B and the normal to the loop can change with time.
- Any combination of the above can occur.



2.2 Some Applications of Faraday's Law



2.2 Faraday's Law

Example 31.1 One Way to Induce an emf in a Coil

A coil consists of 200 turns of wire. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

Example 31.2 An Exponentially Decaying B Field

A loop of wire enclosing an area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of $\mathbf{B}$ varies in time according to the expression $B = B_{max}e^{-at}$, where a is some constant. That is, at $t = 0$ the field is B_{max}, and for $t > 0$, the field decreases

2.2 Faraday's Law

Example 31.2 An Exponentially Decaying B Field

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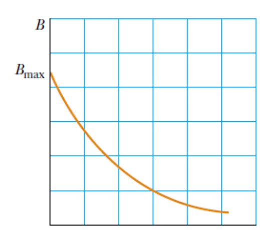


2.3 Motional emf

The electrons in the conductors are affected by two forces, electric and magnetic forces as follows:-

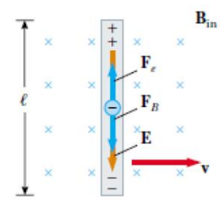
$$F_B = Qv \times B \quad and \quad F_E = QE$$

In equilibrium state the two forces are equal

$$qE = qvB \qquad \text{or} \qquad E = vB$$

Introducing the potential difference in the Equation we get:

$$\Delta V = E\ell = B\ell v$$



2.3 Motional emf

Another derivation

The bar cutting the magnetic field of length l and moves a distance, x

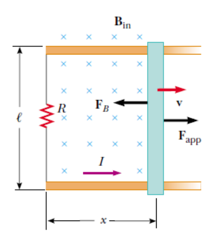
The magnetic flux is then The rate of change of magnetic flux is obtained by first derivative and equal to the negative value of the emf :-

$$\phi_B = Blx$$

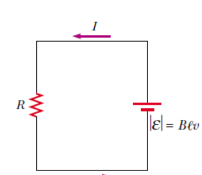
Then

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v$$



(a)



(b)

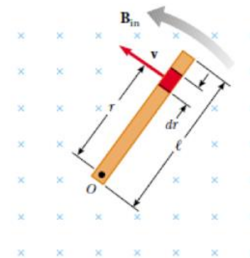
Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$

2.3 Motional emf

Example 31.4 Motional emf Induced in a Rotating Bar

A conducting bar of length ℓ rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation, as shown in Figure 31.11. Find the motional emf induced between the ends of the bar.



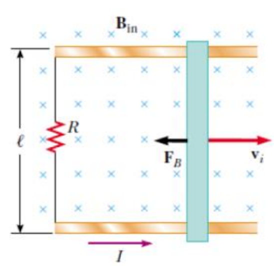
2.3 Motional emf

Example 31.5 Magnetic Force Acting on a Sliding Bar

The conducting bar illustrated in Figure 31.12 moves on two frictionless parallel rails in the presence of a uniform magnetic field directed into the page. The bar has mass m and its length is ℓ. The bar is given an initial velocity $\mathbf{v}_i$ to the right and is released at $t = 0$.

(A) Using Newton's laws, find the velocity of the bar as a function of time.

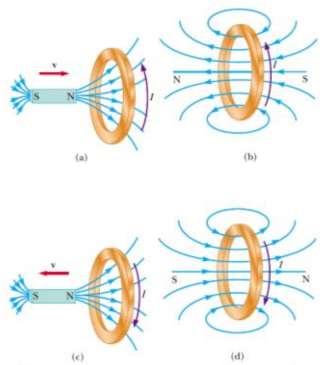
(B) Show that the same result is reached by using an energy approach.

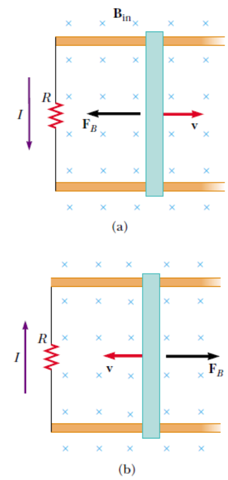


2.4 Lenz law

Lenz law

The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.





2.4 Lenz law

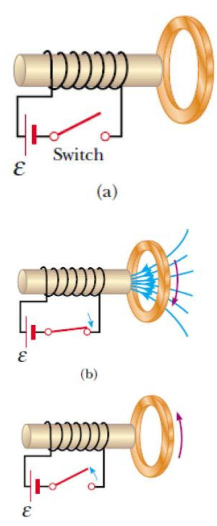
Conceptual Example 31.6 Application of Lenz's Law

A metal ring is placed near a solenoid, as shown in Figure 31.17a. Find the direction of the induced current in the ring

(A) at the instant the switch in the circuit containing the solenoid is thrown closed,

(B) after the switch has been closed for several seconds, and

(C) at the instant the switch is thrown open.



2.4 Lenz law

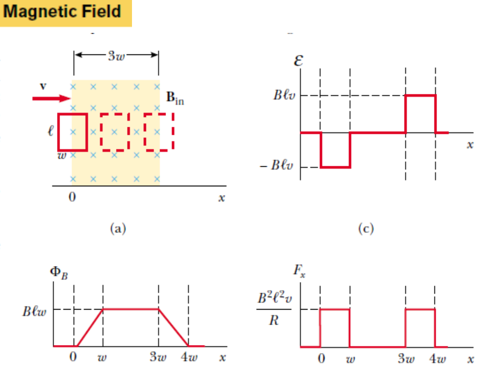
Conceptual Example 31.7 A Loop Moving Through a Magnetic Field

A rectangular metallic loop of dimensions ℓ and w and resistance R moves with constant speed v to the right, as in Figure 31.18a. The loop passes through a uniform magnetic field **B** directed into the page and extending a distance $3w$ along the x axis. Defining x as the position of the right side of the loop along the x axis, plot as functions of x

(A) the magnetic flux through the area enclosed by the loop,

(B) the induced motional emf, and

(C) the external applied force necessary to counter the magnetic force and keep v constant.



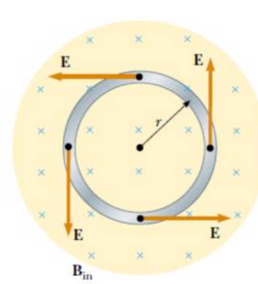
2.4 Induce emf and Electric Field:

"an electric field is created in the conductor as a result of the changing magnetic flux."

We have studied before that the negative rate change in magnetic flux generate an induced ***emf.*** Now the ***emf*** generate an electric field between two points of the potential difference. In a conducting loop of radius ***r*** rotates perpendicular to magnetic field, the electric field is then given by the ***emf*** over the circumstance of the loop. integrated as follows:-

$$q\mathcal{E} = qE(2\pi r)$$

$$E = \frac{\mathcal{E}}{2\pi r}$$



Using this result and the fact that $\Phi_B = BA = \pi r^2 B$ for a circular loop, we find that the induced electric field is

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

2.4 Induce emf and Electric Field:

The emf for any closed path can be expressed as the line integral of $E.ds$ over that path: $\varepsilon = \oint E.ds$.

In general cases, E may not be constant, and the path may not be a circle. Hence, Faraday's law of induction $\varepsilon = -d\Phi_B/dt$, can be written in the general form

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

2.4 Induce emf and Electric Field:

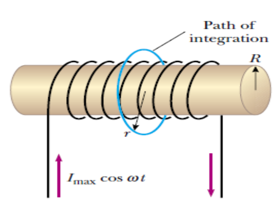
Example 31.8 Electric Field Induced by a Changing Magnetic Field in a Solenoid

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{max} \cos \omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source (Fig. 31.20).

(A) Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis.

Hence, the amplitude of the electric field outside the solenoid falls off as $1/r$ and varies sinusoidally with time.

(B) What is the magnitude of the induced electric field inside the solenoid, a distance r from its axis?



2.6 Maxwell's Equations:

Gauss's law → $\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$

Gauss's law in magnetism → $\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$

Faraday's law → $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$

Ampère–Maxwell law → $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

2.6 Maxwell's Equations:

Gauss's law: the total electric flux through any closed surface equals the net charge inside that surface divided by ε_0.

Gauss's law in magnetism: states that the net magnetic flux through a closed surface is zero.

Faraday's law of induction: This law states that the emf, which is the line integral of the electric field around any closed path, equals the rate of change of magnetic flux through any surface area bounded by that path.

The Ampère–Maxwell law: the line integral of the magnetic field around any closed path is the sum of $\mu_0$times the net current through that path and $\varepsilon_0\mu_0$times the rate of change of electric flux through any surface bounded by that path.

Once the electric and magnetic fields are known at some point in space, the force acting on a particle of charge q :can be calculated from the expression

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

This relationship is called the **Lorentz force law**.