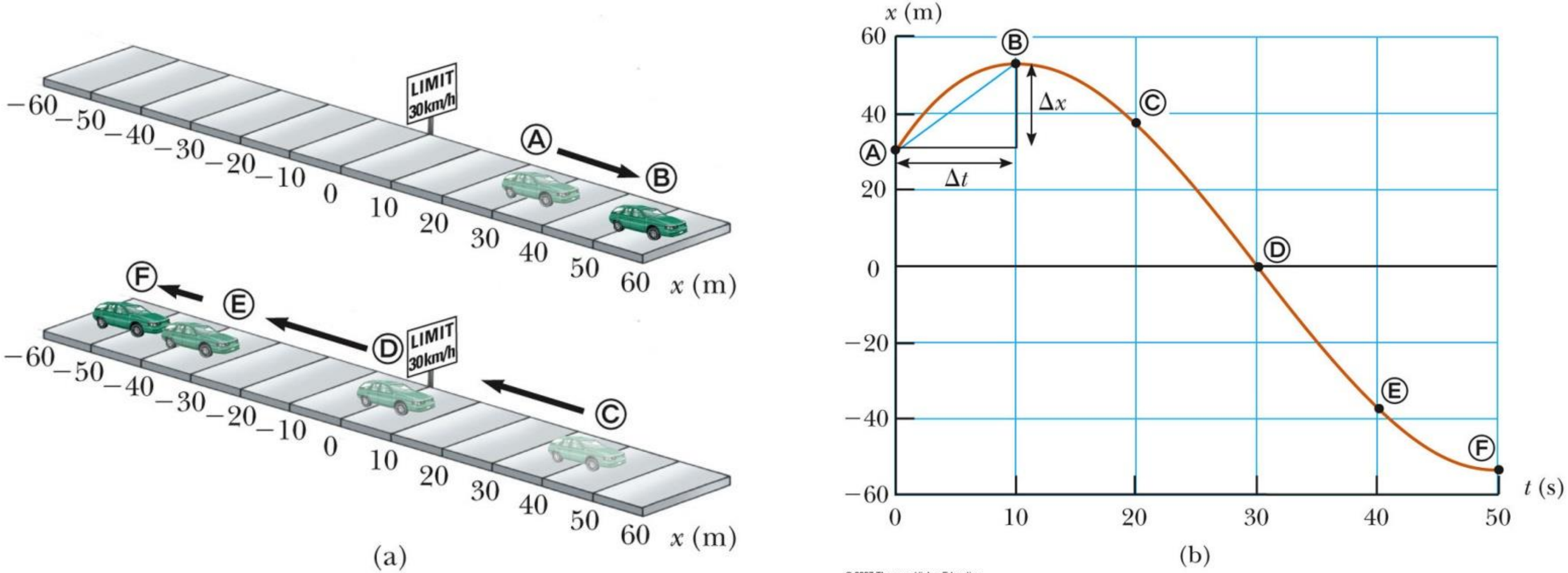
Motion in one Dimension

2.1 Position, Velocity, and Speed:

A particle's **position** *x* is the location of the particle with respect to a chosen reference point that we can consider to be the origin.

- The object's position is its location with respect to a chosen reference point
- Consider the point to be the origin of a coordinate system
- In the diagram, allow the road sign to be the reference point
- The position-time graph shows the motion of the particle (car)
- The smooth curve is a guess as to what happened between the data points



(a) (b)

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2.1 Position, Velocity, and Speed:

- The table gives the actual data collected during the motion of the object (car)
- Positive is defined as being to the right

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

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2.1 Position, Velocity, and Speed:

The **displacement** Δx of a particle is defined as its change in position in some time interval. As the particle moves from an initial position x_i to a final position x_f , its displacement is given by

$$\Delta x \equiv x_f - x_i \qquad (1)$$

- We use the capital Greek letter delta (Δ) to denote the *change* in a quantity.
- Δx is positive if x_f is greater than x_i.
- Δx is negative if x_f is less than x_i.
- SI units are meters (m)
- Displacement is an example of a vector quantity.
- Many other physical quantities, including position, velocity, and acceleration, also are vectors.

In this chapter,

- we use positive(+) and negative (-) signs to indicate vector direction. For example, for horizontal motion the right is the positive direction. It follows that any object always moving to the right undergoes a positive displacement $\Delta x>0$
- Any object moving to the left undergoes a negative displacement so that $\Delta x<0$.

2.1 Position, Velocity, and Speed:

The **average velocity** $v_{x,avg}$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} \qquad (2)$$

where the subscript x indicates motion along the x axis.

The **average speed** v_{avg} of a particle, a scalar quantity, is defined as the total distance d traveled divided by the total time interval required to travel that distance:

$$v_{avg} \equiv \frac{d}{\Delta t} \qquad (3)$$

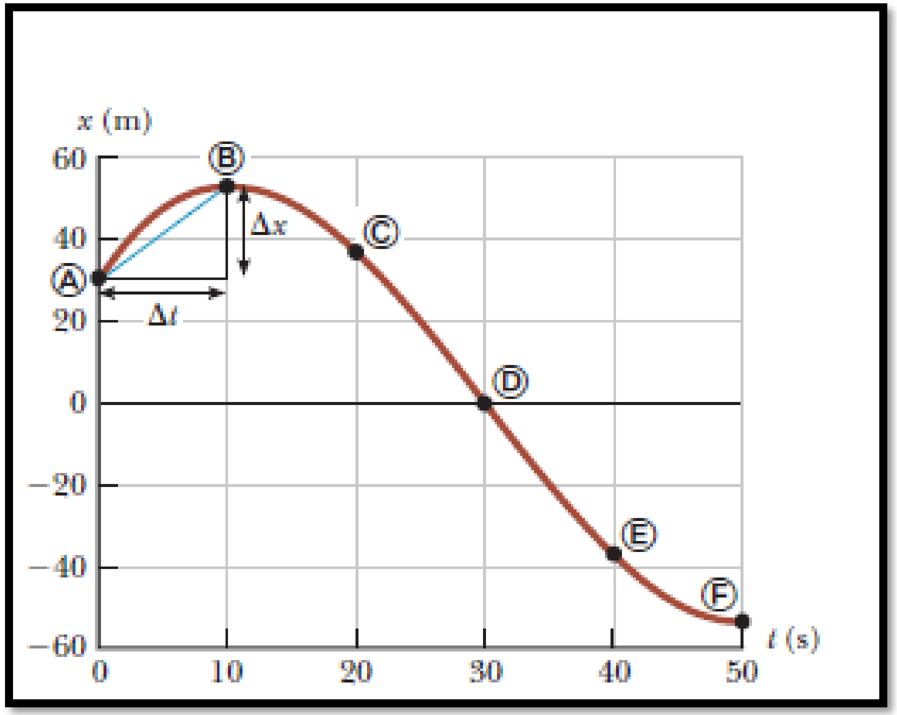
The SI unit of average speed is the same as the unit of average velocity: meters per second.

Unlike average velocity, however, average speed has no direction and is always expressed as a positive number.

2.1 Position, Velocity, and Speed:

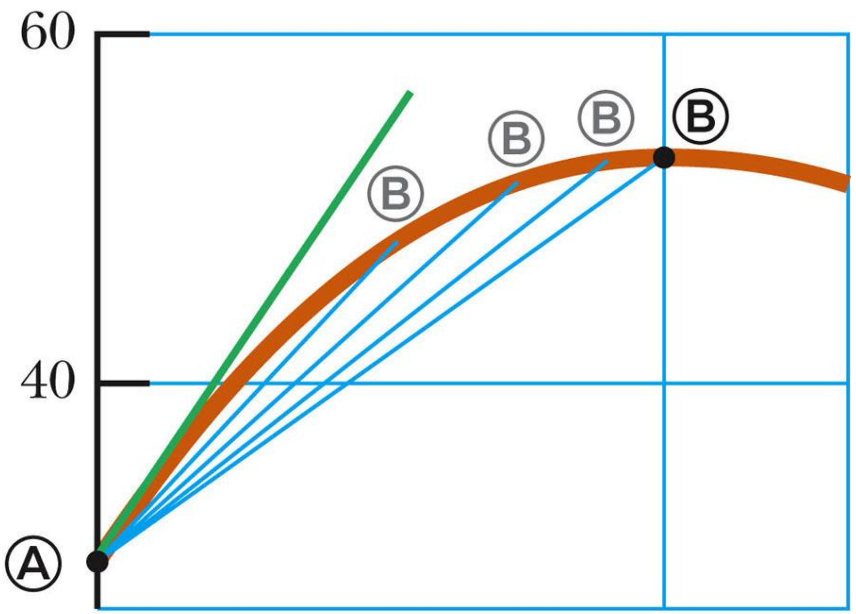
Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Active Figure 2.1a between positions Ⓐ and Ⓕ.



2.2 Instantaneous Velocity and Speed:

- The instantaneous velocity is the slope of the line tangent to the x vs. t curve
- At point A this is the green line
- The light blue lines show that as Δt gets smaller, they approach the green line



The **instantaneous velocity** v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad (4)$$

In calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (5)$$

The instantaneous velocity can be positive, negative, or zero.
The **instantaneous speed** of a particle is defined as the magnitude of its instantaneousvelocity.

2.2 Instantaneous Velocity and Speed:

Conceptual Example 2.2 The Velocity of Different Objects

Consider the following one-dimensional motions: (A) a ball thrown directly upward rises to a highest point and falls back into the thrower's hand; (B) a race car starts from rest and speeds up to 100 m/s; and (C) a spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity has the same value as the average velocity over the entire motion? If so, identify the point(s).

continued

2.2 Instantaneous Velocity and Speed:

Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4a. Because the position of the particle is given by a mathematical function, the motion of the particle is completely known, unlike that of the car in Active Figure 2.1. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

(B) Calculate the average velocity during these two time intervals.

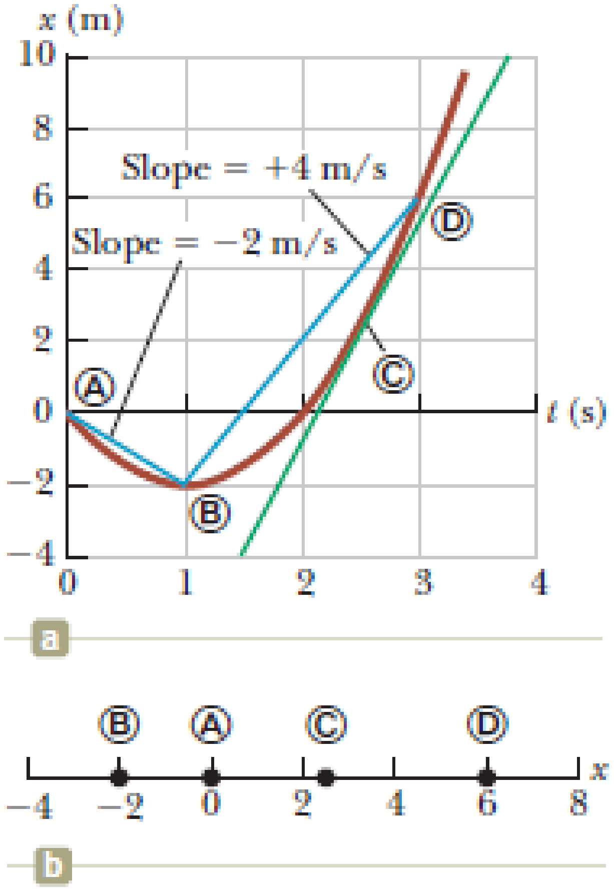


Figure 2.4 (Example 2.3)

Particle Under Constant Velocity

If the velocity of a particle is constant, its instantaneous velocity at any instant during a time interval is the same as the average velocity over the interval. That is, $v_x = v_{x,avg}$.

Therefore, Equation (2) gives us an equation to be used in the mathematical representation of this situation:

$$v_x = \frac{\Delta x}{\Delta t} \qquad (6)$$

Remembering that $\Delta x = x_f - x_i$, we see that $v_x = (x_f - x_i)/\Delta t$, or

$x_f = x_i + v_x \Delta t$

In practice, we usually choose the time at the beginning of the interval to be $t_i = 0$ and the time at the end of the interval to be $t_f = t$, so our equation becomes

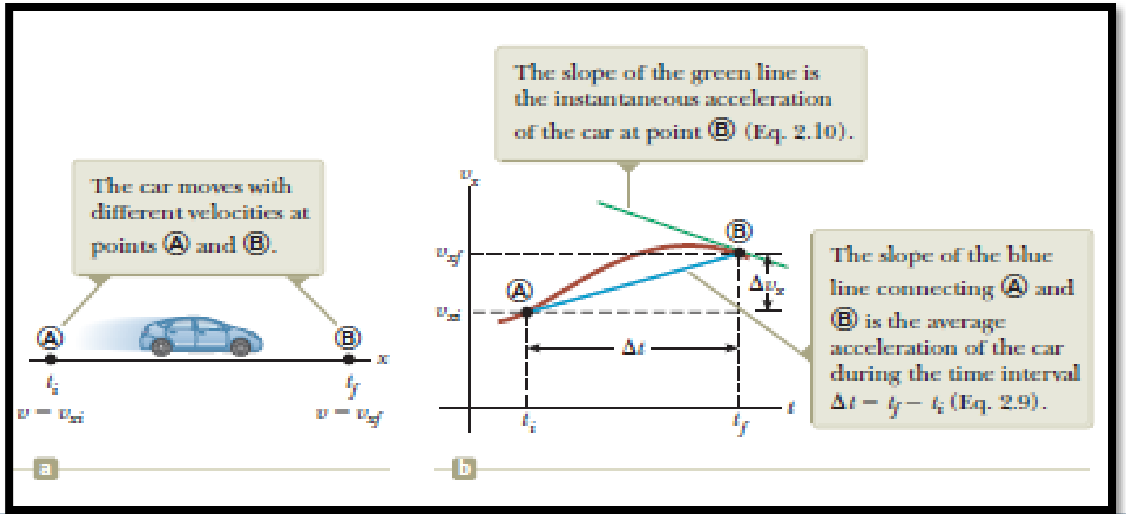
$$x_f = x_i + v_x t \qquad (7)$$

2.3 Acceleration:

The **average acceleration** ***ax, avg of the particle*** is defined as the *change in velocity Δvx divided by the time interval Δt during which* that change occurs:

$$a_{x,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

As with velocity, when the motion being analyzed is one dimensional, we can use positive and negative signs to indicate the direction of the acceleration.

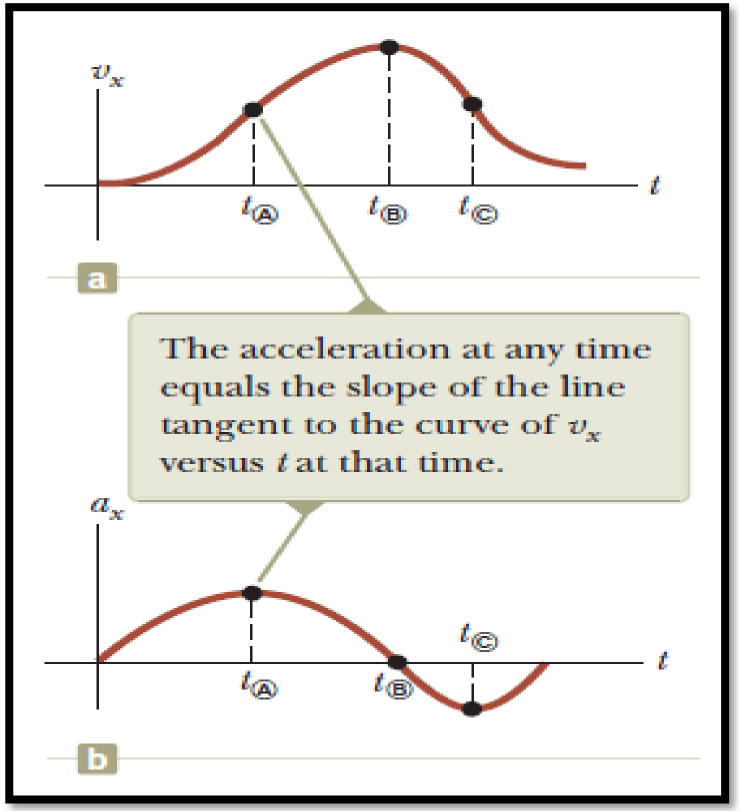


2.3 Acceleration:

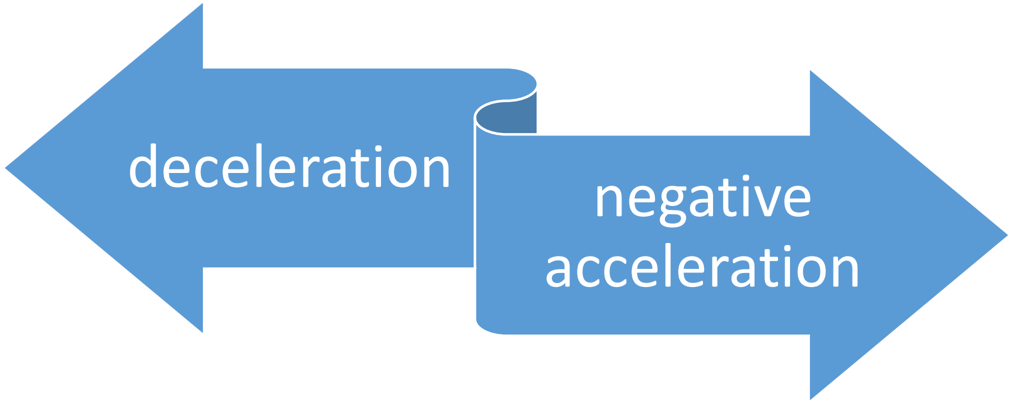
the **instantaneous acceleration** as the limit of the average acceleration as Δt *approaches zero:*

$$\boxed{a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}}$$

When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.



2.3 Acceleration:



	Speeding up (+)	Slowing down (-)
+ velocity	+a	-a
- velocity	-a	+a

2.3 Acceleration:

Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

(B) Determine the acceleration at $t = 2.0$ s.

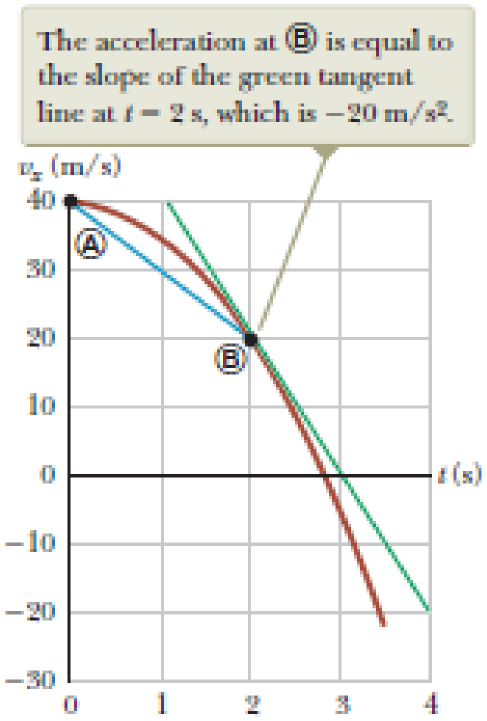
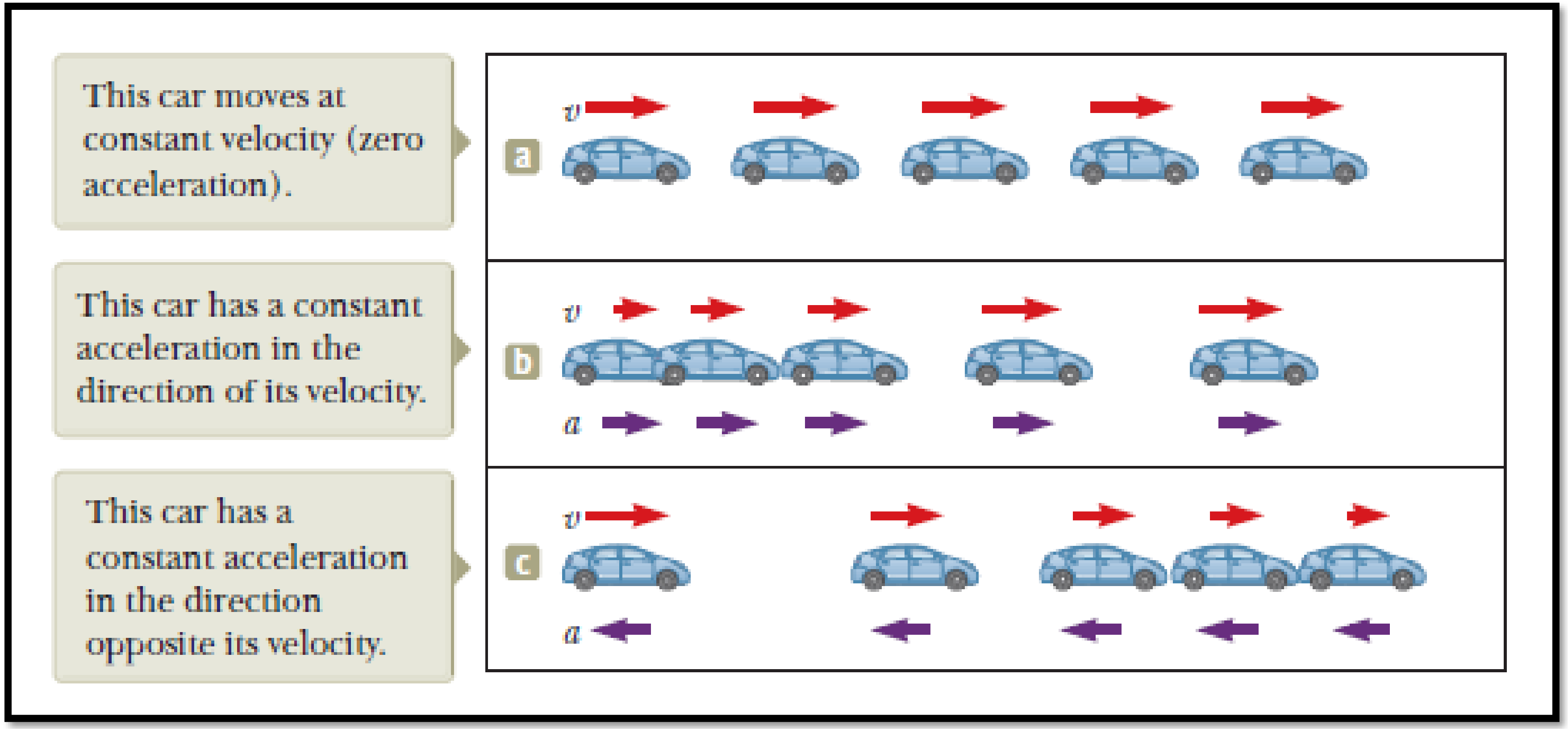


Figure 2.9 (Example 2.6) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = 40 - 5t^2$.

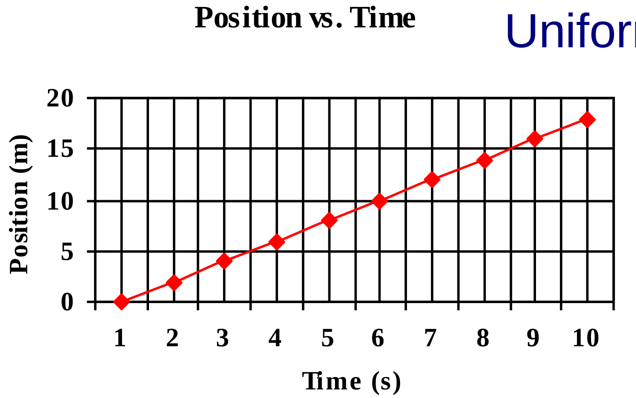
2.4 Motion Diagrams:

motion diagram is sometimes useful to describe the velocity and acceleration while an object is in motion.



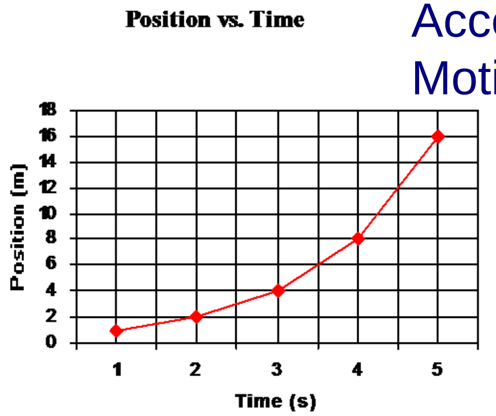
Position. Vs. Time graphs

Uniform Motion

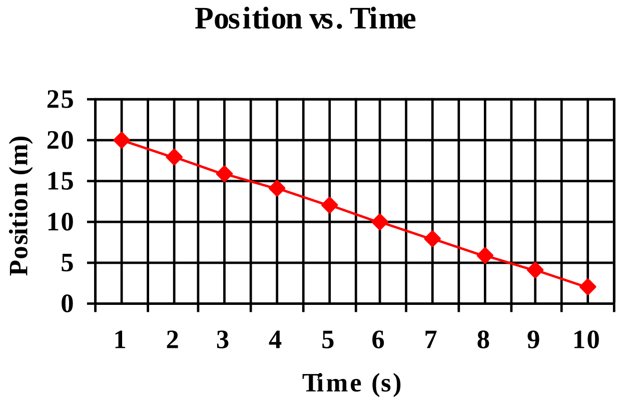


Constant positive velocity
(zero acceleration)

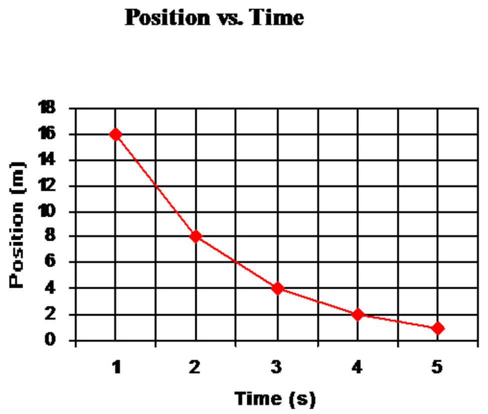
Accelerated Motion



Increasing positive velocity
(positive acceleration)

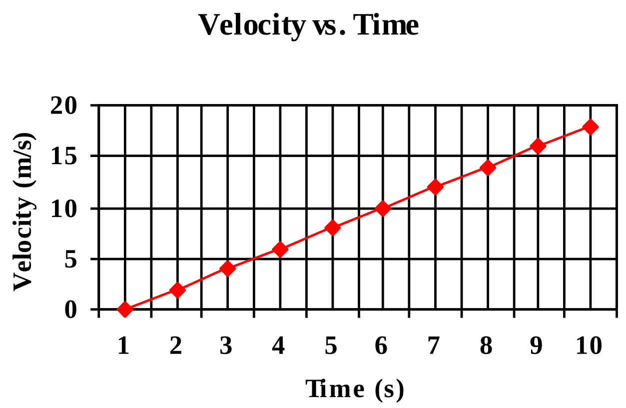


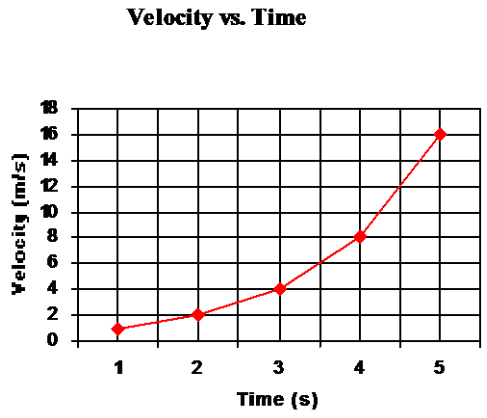
Constant negative velocity
(zero acceleration)

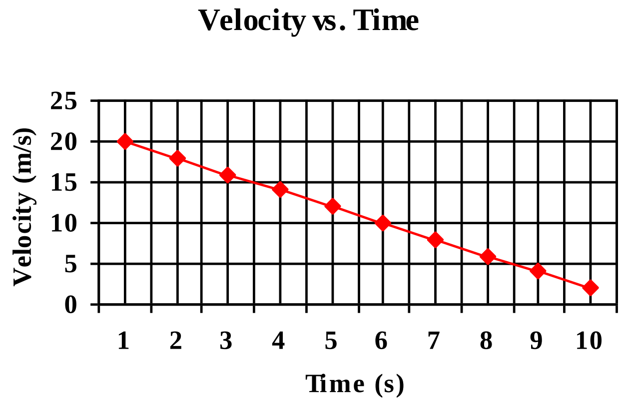


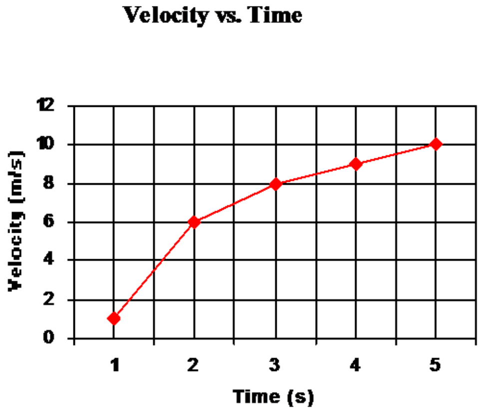
Decreasing negative velocity
(positive acceleration)

Velocity-time graphs

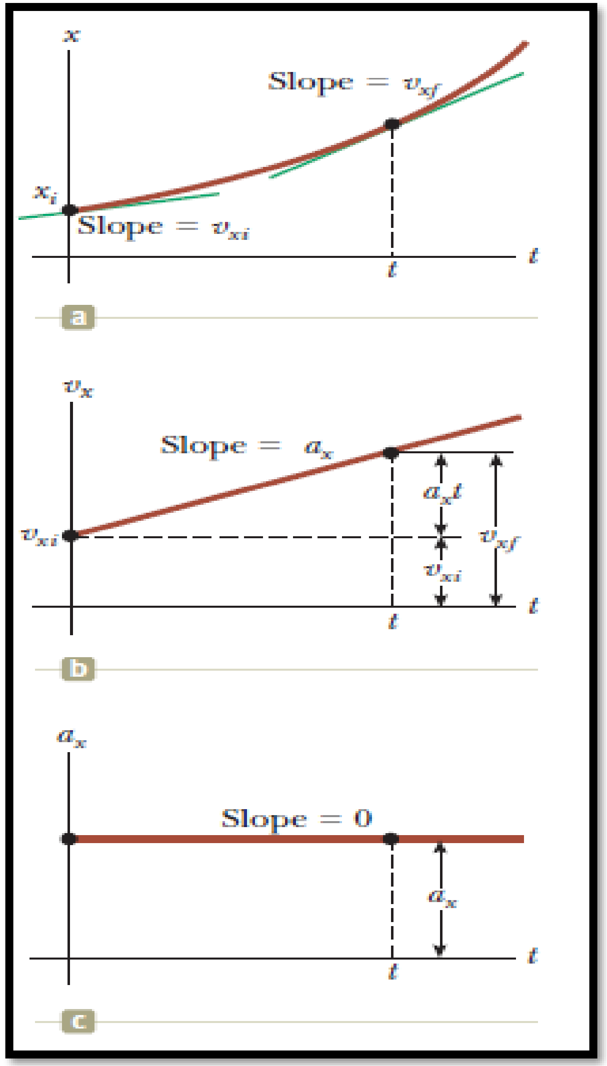








2.6 1D Montion Under Constant Acceleration:



2.6 1D Montion Under Constant Acceleration:

A very simple type of one-dimensional motion is that in which the acceleration is constant. The average acceleration $a_{x,avg}$ over any time interval is equal to the instantaneous acceleration a_x at any instant within the interval.

If we replace $a_{x,avg}$ by a_x in $a_{x,avg} = \frac{\Delta v}{\Delta t}$ and take $t_i = 0$ and t_f to be any later time t, we find that:

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

or

$$v_{xf} = v_{xi} + a_x t \quad (for\ constsnt\ a_x) \qquad (1)$$

2.6 1D Montion Under Constant Acceleration:

we can express the average velocity in any time interval as the arithmetic mean of the initial velocity vxi and the final velocity vxf :

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2} \; (for\ constsnt\ a_x)$$

Notice that this expression for average velocity applies *only* if the acceleration is constant.

To obtain the position of an object as a function of time, Recalling that Δx in $v_{x,avg} \equiv \frac{\Delta x}{\Delta t}$ represents $xf - xi$ and recognizing that $\Delta t = tf - ti = t - 0 = t,$ we find that:

$$x_f - x_i = v_{x,avg}t = \frac{1}{2}(v_{xi} + v_{xf})t$$

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t \;\; (for\ constant\ a_x) \qquad (2)$$

2.6 1D Montion Under Constant Acceleration:

We can obtain another useful expression for the position of a particle under constant acceleration by substituting Equation (1) into Equation (2):

$$x_f = x_i + \frac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad (for\ constant\ a_x) \quad (3)$$

Finally, we can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from Equation (1) into Equation (2):

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right)$$

$$v_f^2 = v_i^2 + 2a_x(x_f - x_i) \quad (for\ constant\ a_x) \quad (4)$$

2.6 1D Montion Under Constant Acceleration:

For motion at *zero* acceleration, we see from Equations (1) and (3) that:

$$\left.\begin{aligned} v_{xf} = v_{xi} = v_x \\ x_f = x_i + v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when the acceleration of a particle is zero, its velocity is constant and its position changes linearly with time.

TABLE 2.2 *Kinematic Equations for Motion of a Particle Under Constant Acceleration*

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

2.6 1D Montion Under Constant Acceleration:

Example 2.7 Carrier Landing

A jet lands on an aircraft carrier at a speed of 140 mi/h ($\approx$ 63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

(B) If the jet touches down at position $x_i = 0$, what is its final position?

2.6 1D Montion Under Constant Acceleration:

Example 2.8 Watch Out for the Speed Limit!

A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s^2. How long does it take her to overtake the car?

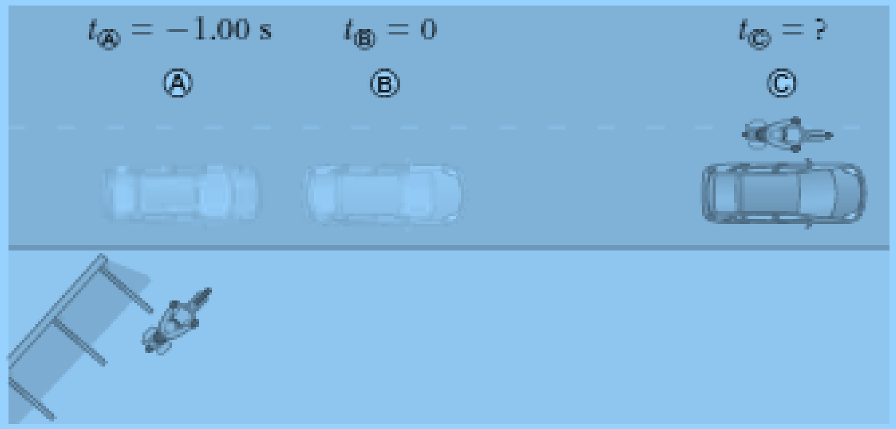


Figure 2.13 (Example 2.8) A speeding car passes a hidden trooper.

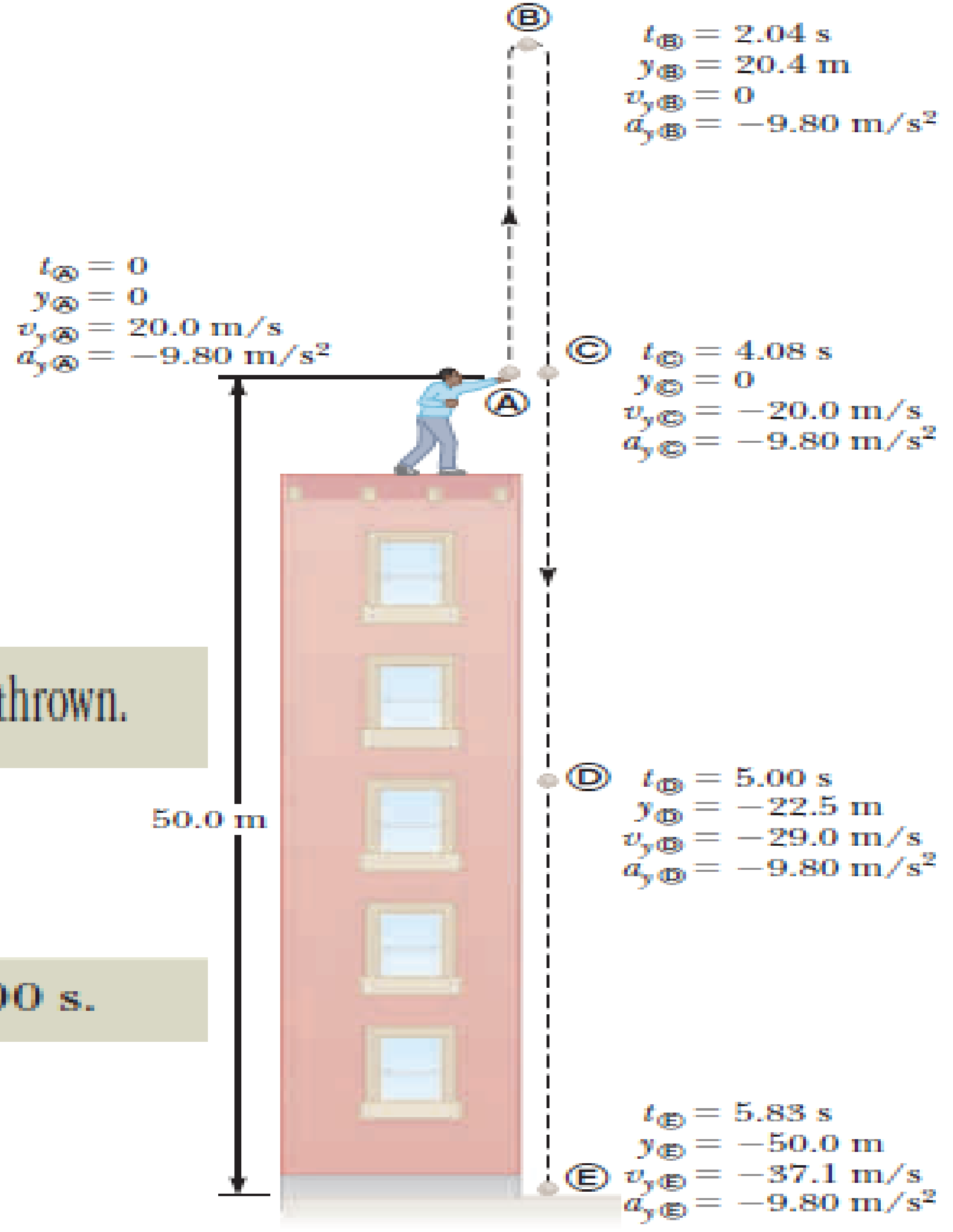
2.7 Free Fall

- In the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity.
- When we use the expression *freely falling object,* we do not necessarily refer to an object dropped from rest.
- Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences an acceleration directed *downward*, regardless of its initial motion.
- We shall denote the magnitude of the *free-fall acceleration* by the symbol *g.*
- The only modification for freely falling objects that we need to make in these equations is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal direction(x) and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Therefore, we choose a_y = -g =$-$9.80 m/s^2, where the negative sign means that the acceleration of a freely falling object is downward.

Example 2.10 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in Figure 2.14.

(A) Using $t_{Ⓐ} = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine the time at which the stone reaches its maximum height.



(B) Find the maximum height of the stone.

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

(D) Find the velocity and position of the stone at $t = 5.00$ s.