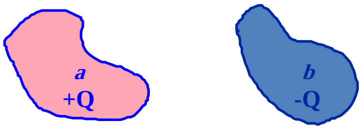
Chapter 8
Capacitance

26.1 Definition of Capacitance

Capacitor: two conductors (separated by an insulator) usually oppositely charged



The capacitance, C, of a capacitor is defined as a ratio of the magnitude of a charge on either conductor to the magnitude of the potential difference between the conductors

$$C = \frac{Q}{\Delta V}$$

Note that by definition capacitance is always a positive quantity. Because positive and negative charges are separated in the system of two conductors in a capacitor, there is electric potential energy stored in the system.

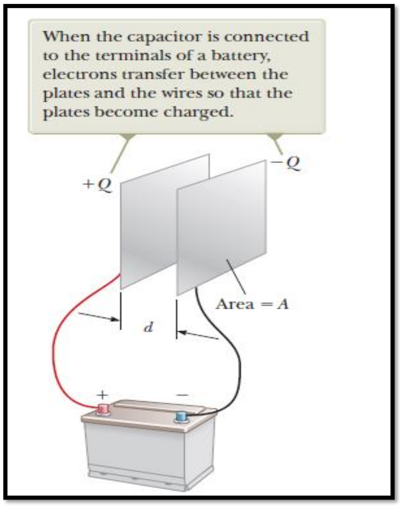
26.1 Definition of Capacitance

The unit of C is the farad (F), but most capacitors have values of C ranging from picofarads to microfarads (pF to μF).

$$1\,F = 1\,C/V$$

Recall, micro ⇒10^{-6}, nano ⇒10^{-9}, pico ⇒10^{-12}

If the external potential is disconnected, charges remain on the plates, so capacitors are good for storing charge (and energy).



When the capacitor is connected to the terminals of a battery, electrons transfer between the plates and the wires so that the plates become charged.

26.2 Calculating Capacitance

Parallel-Plate Capacitors:

Two parallel metallic plates of equal area A are separated by a distance d. One plate carries a charge Q , and the other carries a charge -Q .

The surface charge density on each plate is σ=Q/A. If the plates are very close together (in comparison with their length and width), we can assume the electric field is uniform between the plates and zero elsewhere. The value of the electric field between the plates is

$$\boxed{E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}}$$

26.2 Calculating Capacitance

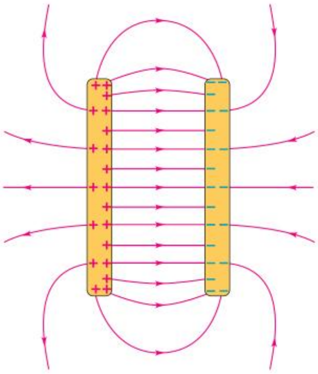
Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals Ed; therefore,

$$\boxed{\Delta V = Ed = \frac{Qd}{\epsilon_0 A}}$$

Substituting this result into C=Q/ΔV, we find that the capacitance is

$$\boxed{C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$



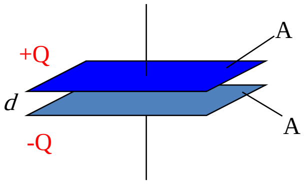
26.2 Calculating Capacitance

The capacitance of a device depends on the geometric arrangement of the conductors

$$C = \varepsilon_0 \frac{A}{d}$$

where A is the area of one of the plates, d is the separation, ε_0 is a constant called the permittivity of free space,

$\varepsilon_0 = 8.85'10^{-12}$ $C^2/N \cdot m^2$



$$k_e = \frac{1}{4\pi\varepsilon_0}$$

26.2 Calculating Capacitance

A parallel plate capacitor has plates 2.00 m^2 in area, separated by a distance of 5.00 mm. A potential difference of 10,000 V is applied across the capacitor. Determine

a) the capacitance?

b) the charge on each plate?

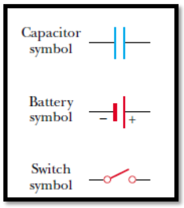
26.2 Calculating Capacitance

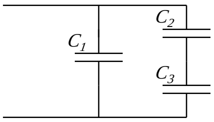
imagine a spherical charged conductor. The electric field lines around this conductor are exactly the same as if there were a conducting shell of infinite radius, concentric with the sphere and carrying a charge of the same magnitude but opposite sign. Thus, we can identify the imaginary shell as the second conductor of a two-conductor capacitor. We now calculate the capacitance for this situation. The electric potential of the sphere of radius R is simply keQ/R, and setting V = 0 for the infinitely large shell, we have:

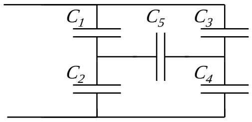
$$\boxed{C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/R} = \frac{R}{k_e} = 4\pi\epsilon_0 R}$$

26.3 Combinations of Capacitors

- Capacitors in circuits symbols
- It is very often that more than one capacitor is used in an electric circuit
- We would have to learn how to compute the equivalent capacitance of certain combinations of capacitors







26.3 Combinations of Capacitors

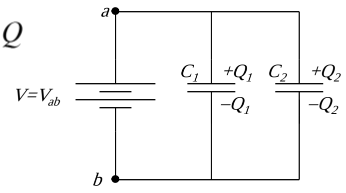
a. Parallel combination

Connecting a battery to the parallel combination of capacitors is equivalent to introducing the same potential difference for both capacitors,

$$V_1 = V_2 = V$$

A total charge transferred to the system from the battery is the sum of charges of the two capacitors,

$$Q_1 + Q_2 = Q$$



26.3 Combinations of Capacitors

Substituting these three relationships for charge into the Equation for charge ,we have

$$Ceq\ \Delta V= C1\Delta V+C2\Delta V$$

$$Ceq = C1+C2 \qquad \text{(parallel combination)}$$

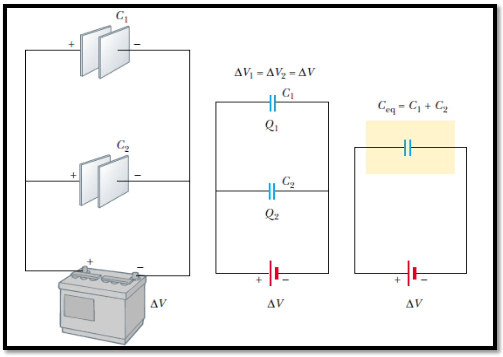
If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

$$Ceq = C1+C2 +C3+\ldots\ldots\ldots$$

It follows that the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors

26.3 Combinations of Capacitors

A 3 μF capacitor and a 6 μF capacitor are connected in parallel across an 18 V battery. Determine the equivalent capacitance and total charge deposited.



26.3 Combinations of Capacitors

b. Series combination

Connecting a battery to the serial combination of capacitors is equivalent to introducing the same charge for both capacitors,

$$Q_1 = Q_2 = Q$$

A voltage induced in the system from the battery is the sum of potential differences across the individual capacitors,

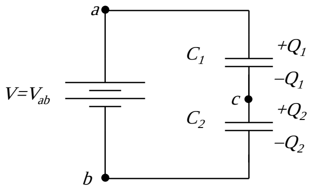
$$V = V_1 + V_2$$

$$Q_1 = C_1V_1 \quad Q_2 = C_2V_2$$

$$\frac{1}{C_{eq}} \equiv \frac{V}{Q} = \frac{V_1+V_2}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{V_1}{Q_1} + \frac{V_2}{Q_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad Q_1 = Q_2 = Q$$

$$V_1 + V_2 = V$$



26.3 Combinations of Capacitors

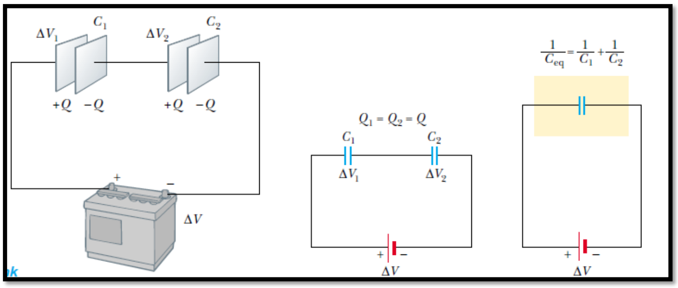
Analogous formula is true for any number of capacitors,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \quad \text{(series combination)}$$

It follows that the equivalent capacitance of a series combination of capacitors is always less than any of the individual capacitance in the combination

26.3 Combinations of Capacitors

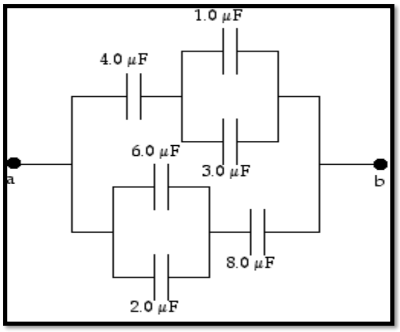
A 3 μF capacitor and a 6 μF capacitor are connected in series across an 18 V battery. Determine the equivalent capacitance.



26.3 Combinations of Capacitors

Example 26.4 Equivalent Capacitance

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.



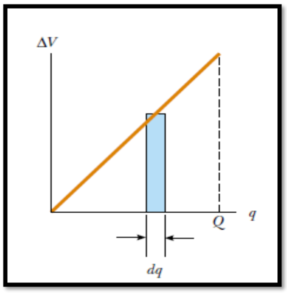
16.3 Energy stored in a charged capacitor

Consider a battery connected to a capacitor
A battery must do work to move electrons from one plate to the other. The work done to move a small charge Δ q across a voltage V is

$$\Delta W = V \Delta q.$$

As the charge increases, V increases so the work to bring Δ q increases.
Using calculus we find that the energy (U) stored on a capacitor is given by:

$$U = \frac{1}{2} QV = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$



16.3 Energy stored in a charged capacitor

Find electric field energy density (energy per unit volume) in a parallel-plate capacitor

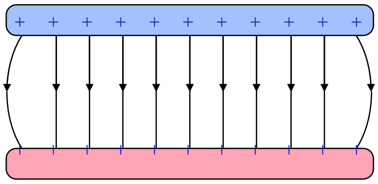
$$U = \frac{1}{2} CV^2$$

$$C = \frac{\varepsilon_0 A}{d} \quad volume = Ad \quad V = Ed$$

$$u \equiv U / volume = \text{energy density}$$

$$= \frac{1}{2} \frac{\varepsilon_0 A}{d} (Ed)^2 / (Ad)$$

$$u = \frac{1}{2} \varepsilon_0 E^2$$



16.3 Energy stored in a charged capacitor

In the circuit shown V = 48V, C1 = 9mF, C2 = 4mF and C3 = 8mF.

(a) determine the equivalent capacitance of the circuit,

(b) determine the energy stored in the combination by calculating the energy stored in the equivalent capacitance.