Chapter 24
Electric potential

25.1 Potential Difference and Electric Potential

When a test charge q0 is placed in an electric field E created by some source charge distribution, the electric force acting on the test charge is F=q0E. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement.

The work done by the electric field on the charge is $\boldsymbol{F}.d\boldsymbol{s} = q_0\boldsymbol{E}.d\boldsymbol{s}$. As this amount of work is done by the field, the potential energy of the charge–field system is changed by an amount $dU = -q_0\boldsymbol{E}.d\boldsymbol{s}$.

25.1 Potential Difference and Electric Potential

For a finite displacement of the charge from point A to point B, the change in potential energy of the system $\Delta U = UB - UA$ is

$$\boxed{\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s}}$$

Where ds is an infinitesimal displacement , E is the electric field and q0 is the test charge.

25.1 Potential Difference and Electric Potential

The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a value at every point in an electric field, is called the electric potential (or simply the potential) V.

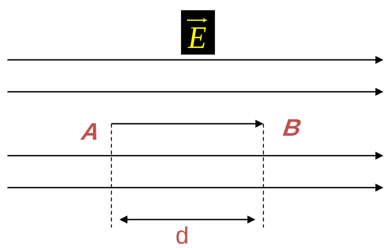
$$\boxed{V = \frac{U}{q_0}}$$

Thus, the electric potential at any point in an electric field is

Potential energy is a scalar quantity, and The electric potential also is a scalar quantity.

25.1 Potential Difference and Electric Potential

If the test charge is moved between two positions A and B in an electric field



- The electrostatic force is conservative.
- As in mechanics, work is:

$$W = F\ d \cos \theta$$

- Work done on the positive charge by moving it from A to B:

$$W = F\ d \cos \theta = (q\ E)\ d$$

25.1 Potential Difference and Electric Potential

The potential difference $\Delta V = V_B - V_A$ between two points A and B in an electric field is defined as the change in potential energy of the system when a test charge is moved between the points divided by the test charge q_0 :

$$\boxed{\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}}$$

The potential difference between A and B depends only on the source charge distribution (consider points A and B without the presence of the test charge), while the difference in potential energy exists only if a test charge is moved between the points.

25.1 Potential Difference and Electric Potential

The work done by an external agent in moving a charge q through an electric field at constant velocity is

$$\boxed{W = q\Delta V}$$

The SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):

$$\boxed{1\ \mathrm{V} = 1\ \frac{\mathrm{J}}{\mathrm{C}}}$$

That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

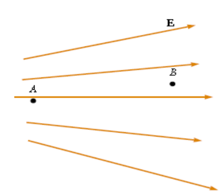
Quiz

2: In the Figure, a negative charge is placed at A and then moved to B. The change in potential energy of the charge–field system for this process is

(a) positive,

(b) negative,

(c) zero.



1: In the Figure, two points A and B are located within a region in which there is an electric field. The potential difference $\Delta V = V_B - V_A$ is:

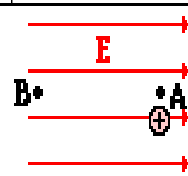
(a) positive

(b) negative

(c) zero

POTENTIAL ENERGY IN A UNIFORM FIELD

	+ Charge	- Charge
Along E	Loses PEe	Gains PEe
Opposite E	Gains PEe	Loses PEe

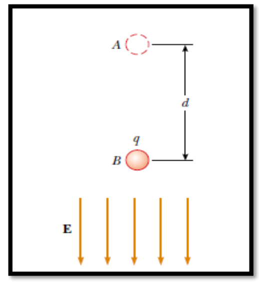


Example 1:

An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37 x 10^{-17} J. Calculate the charge on the ion.

25.2 Potential Differences in a Uniform Electric Field

Consider a uniform electric field directed along the negative y axis, as shown in Figure a. Let us calculate the potential difference between two points A and B separated by a distance , magnitude |s| = *d* , where s is parallel to the field lines.



$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B (E \cos 0°)\, ds = -\int_A^B E\, ds$$

25.2 Potential Differences in a Uniform Electric Field

- Because E is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E \int_A^B ds = -Ed$$

- The negative sign indicates that the electric potential at point B is lower than at point A; that is, $V_B < V_A$.
- Electric field lines always point in the direction of decreasing electric potential.

25.2 Potential Differences in a Uniform Electric Field

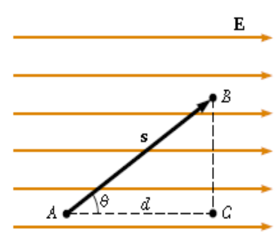
- Now suppose that a test charge q0 moves from A to B. We can calculate the change in the potential energy of the charge–field system:

$$\boxed{\Delta U = q_0 \Delta V = -q_0 E d}$$

- From this result, we see that if q0 is positive, then ΔU is negative. We conclude that a system consisting of a positive charge and an electric field loses electric potential energy when the charge moves in the direction of the field.

25.2 Potential Differences in a Uniform Electric Field

Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector s is not parallel to the field lines, as shown in the Figure below.



A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the same electric potential.

Then

$$\boxed{\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\mathbf{E} \cdot \int_A^B d\mathbf{s} = -\mathbf{E} \cdot \mathbf{s}}$$

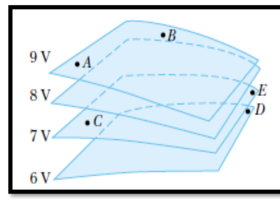
The change in potential energy of the charge–field system is

$$\boxed{\Delta U = q_0 \Delta V = -q_0 \mathbf{E} \cdot \mathbf{s}}$$

25.2 Potential Differences in a Uniform Electric Field

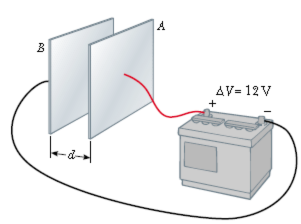
We conclude that all points in a plane perpendicular to a uniform electric field are at the same electric potential.

The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.



Example 2 :

A battery produces a specified potential difference ΔV between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. **Find the magnitude of the electric field between the plates.**



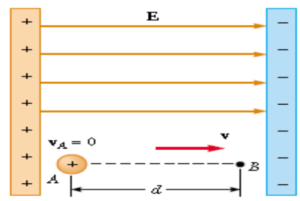
Example 3:Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of 8.0 x10^4 V/m . The proton undergoes a displacement of 0.50 m in the direction of E.

(A) Find the change in electric potential between points A and B.

(B) Find the change in potential energy of the proton–field system for this displacement.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.



25.3 Electric Potential and Potential Energy Due to Point Charges

To find the electric potential at a point located a distance r from the charge, we begin with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

where A and B are the two arbitrary points. At any point in space, the electric field due to the point charge is $\mathbf{E} = k_e q \hat{\mathbf{r}} / r^2$

where rˆ is a unit vector directed from the charge toward the point.

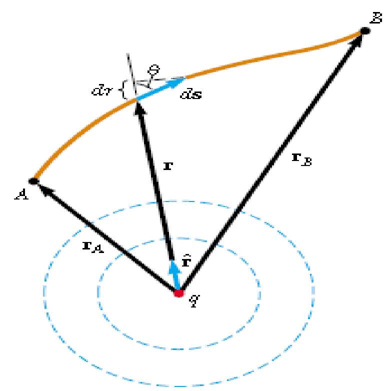
The quantity E . ds can be expressed as

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s}$$

25.3 Electric Potential and Potential Energy Due to Point Charges

Because the magnitude of $\hat{\mathbf{r}}$ is 1, the dot product $\hat{\mathbf{r}}\,.d\mathbf{s} = ds\cos\theta$, where θ is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$.

Furthermore, $ds \cos\theta$ is the projection of $d\mathbf{s}$ onto $\mathbf{r}$; thus, $ds \cos\theta = dr$.



25.3 Electric Potential and Potential Energy Due to Point Charges

Making these substitutions, we find that $\mathbf{E}.\,d\mathbf{s} = (k_e\, q/r^2)dr$; hence, the expression for the potential difference becomes:

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\Big]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

This equation shows us that the integral of $\mathbf{E}.\,d\mathbf{s}$ is *independent* of the path between points *A* and *B*.

It is customary to choose the reference of electric potential for a point charge to be V = 0 at rA = ∞. With this reference choice, the electric potential created by a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r}$$

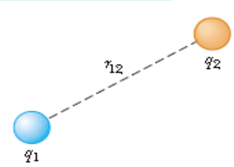
25.3 Electric Potential and Potential Energy Due to Point Charges

For a group of point charges, we can write the total electric potential at P in the form

$$\boxed{V = k_e \sum_i \frac{q_i}{r_i}}$$

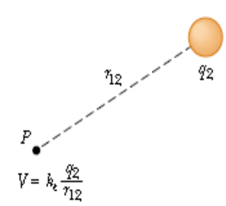
where the potential is again taken to be zero at infinity and r_i is the distance from the point P to the charge q_i .

25.3 Electric Potential and Potential Energy Due to Point Charges



We now consider the potential energy of a system of two charged particles. If $V2$ is the electric potential at a point P due to charge $q2$, then the work an external agent must do to bring a second charge $q1$ from infinity to P without acceleration is $q1\,V2$.
Therefore, we can express the potential energy of the system as

$$\boxed{U = k_e \frac{q_1 q_2}{r_{12}}}$$



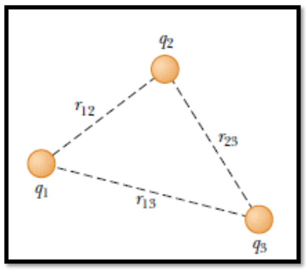
Note that if the charges are of the same sign, U is positive. This is consistent with the fact that positive work must be done by an external agent on the system to bring the two charges near one another (because charges of the same sign repel).

If the charges are of opposite sign, U is negative; this means that negative work is done by an external agent against the attractive force between the charges of opposite sign as they are brought near each other—a force must be applied opposite to the displacement to prevent q_1 from accelerating toward q_2 .

25.3 Electric Potential and Potential Energy Due to Point Charges

The total potential energy of the system of three charges

$$U = k_e\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

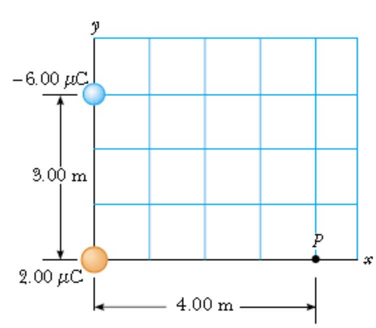


Example 4 :

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure a.

(A)Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Fig. b).



(a)