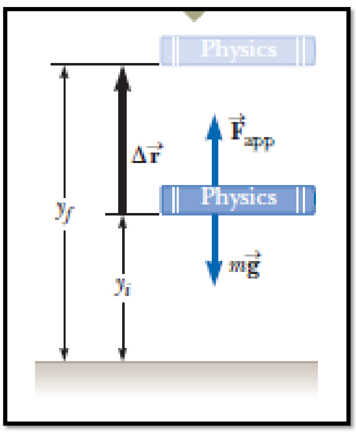
Potential energy

8.1 Potential Energy of a System

Let us consider a mass (m) at an initial height (y_i) above the ground to a final height y_f as in Figure.

then the only force that does work on the brick as it falls is the gravitational force exerted on the mass (mg) .

The work Wg done by the gravitational force as the mass undergoes a downward displacement ($\Delta\vec{r} = \Delta y\hat{j}$) is



$$W_{ext} = (\vec{F}_{app}) \cdot (\Delta\vec{r}) = (mg\hat{j}\,) \cdot [(\,y_f - y_i)\hat{j}] = mgy_f - mgy_i \qquad (1)$$

8.1 Potential Energy of a System

The **gravitational potential energy** U_g:

$$U_g = mgy \qquad (2)$$

- The units of gravitational potential energy are joules, the same as the units of work and kinetic energy.
- Potential energy, like work and kinetic energy, is a scalar quantity.

Using our definition of gravitational potential energy, Equation (1) can now be rewritten as:

$$W_{ext} = \Delta U_g$$

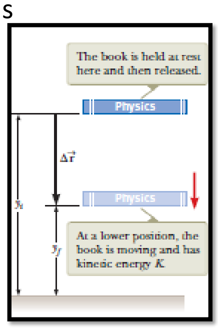
which describes that the net external work done on the system in this situation appears as a change in the gravitational potential energy of the system.

8.2 Analysis Model: Isolated System (Energy)

- Gravitational potential energy depends only on the vertical height of the object above the surface of the Earth.
- In solving problems, you must choose a reference configuration for which the gravitational potential energy of the system is set equal to some reference value, which is normally zero.

The work done *on the book alone* by the gravitational force (Fig.) as the book falls back to its original height. As the book falls from *yi* to *yf* , the work done by the gravitational force on the book is

$$W_{on\ book} = (m\vec{g}) \cdot (\Delta\vec{r}) = (-mg\hat{j}) \cdot [(y_f - y_i)\hat{j}] = mgy_i - mgy_f$$



From the work–kinetic energy theorem , the work done on the book is equal to the change in the kinetic energy of the book:

$$W_{on\ book} = \Delta K_{book}$$

We can equate these two expressions for the work done on the book:

$$\Delta K_{book} = mgy_i - mgy_f \quad (3)$$

8.2 Analysis Model: Isolated System (Energy)

Let us now relate each side of this equation to the *system* of the book and the Earth. For the right-hand side,

$$mgy_i - mgy_f = -(mgy_f - mgy_i) = -\Delta U_g$$

where $U_g = mgy$ is the gravitational potential energy of the system. For the left-hand side of Equation (3), because the book is the only part of the system that is moving, we see that $\Delta K_{book} = \Delta K$, where K is the kinetic energy of the system. Therefore, with each side of Equation (3) replaced with its system equivalent, the equation becomes

$$\Delta K = -\Delta U_g \qquad (4)$$

8.2 Analysis Model: Isolated System (Energy)

We move the change in potential energy to the left side of the equation:

$$\Delta K + \Delta U_g = 0 \qquad (5)$$

The left side represents a sum of changes of the energy stored in the system. The right-hand side is zero because there are no transfers of energy across the boundary of the system; the book–Earth system is *isolated* from the environment.

$$E_{mec} = K + U_g$$

Let us now write the changes in energy in Equation (5) explicitly:

$$K_f + U_f = K_i + U_i \qquad (6)$$

The total mechanical energy is conserved and remains the same at all times

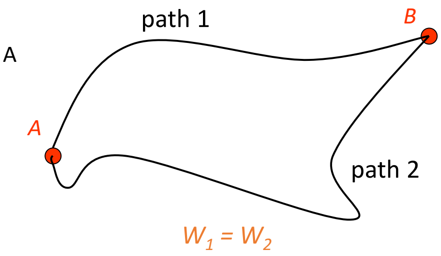
$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

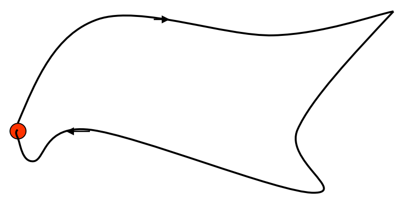
8.3 Conservative and Nonconservative Forces:

Conservative Forces
A force is called "conservative" if the work done (in going from A to B) is *the same for all paths* from A to B.

An equivalent definition:

For a conservative force, the work done on any ***closed path*** is zero.





* The work depends only upon the initial and final positions of the object
* Any conservative force can have a potential energy function associated with it
Work done by gravity

$$W_g = U_i - U_f = mgy_i - mgy_f$$

8.3 Conservative and Nonconservative Forces:

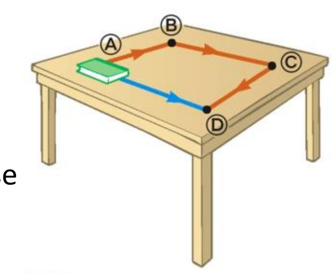
A force is nonconservative if the work it does on an object depends on the path taken by the object between its final and starting points.

- The work depends upon the movement path
- For a non-conservative force, potential energy can NOT be defined
- As the book moves through a distance d, the only force that does work on it is the force of kinetic friction.
- This force causes a decrease in the kinetic energy of the book. This decrease was calculated

$$\Delta K = -f_k d$$

- the mechanical energy of the system changes because of the force of kinetic friction

$$\Delta E_{mech} = \Delta K + \Delta U_g = -f_k d$$



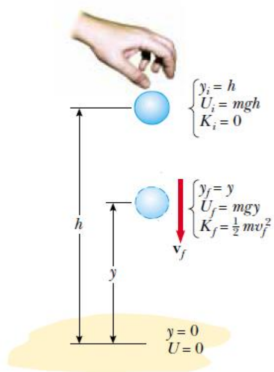
8.2 Analysis Model: Isolated System (Energy)

Example 8.2 Ball in Free Fall

A ball of mass m is dropped from a height h above the ground, as shown in Figure 8.6.

(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

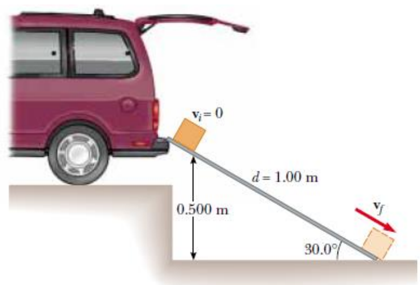
(B) Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.



8.4 Changes in Mechanical Energy for Nonconservative Forces

Example 8.6 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0°, as shown in Figure 8.11. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.



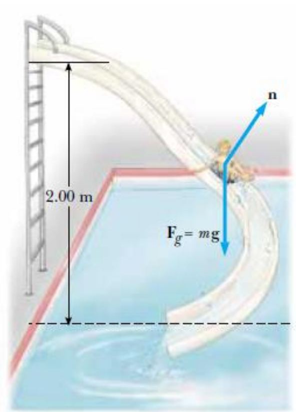
8.4 Changes in Mechanical Energy for Nonconservative Forces

Example 8.7 Motion on a Curved Track

A child of mass m rides on an irregularly curved slide of height $h = 2.00$ m, as shown in Figure 8.12. The child starts from rest at the top.

(A) Determine his speed at the bottom, assuming no friction is present.

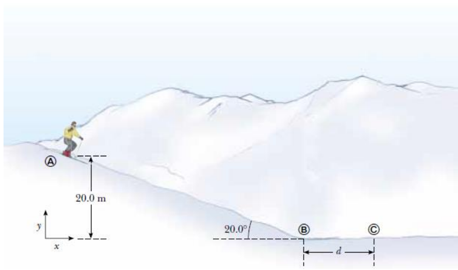
(B) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that $v_f = 3.00$ m/s and $m = 20.0$ kg.



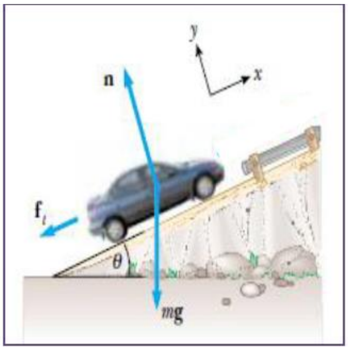
8.4 Changes in Mechanical Energy for Nonconservative Forces

Example 8.8 Let's Go Skiing!

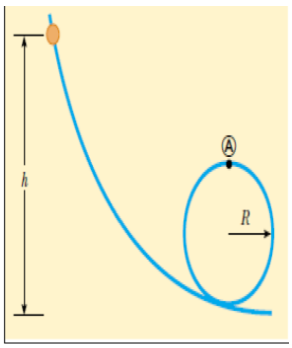
A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure 8.13. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop?



Consider a car of mass 30 kg that is slide 5m and it is at angle $\theta = 20$ with respect to the ground , starts from rest at the bottom of the slide. The $\mu_k = 0.2$
What is the total work done by the friction force on the car ?



5 - A bead slides without friction around a loop-the-loop
The bead is released from a height $h = 3.50R$.
(a) What is its speed at point (A)
(b) How large is the normal force on it if its mass is 5.00 g?



24 - A particle of mass m = 5kg is released from point A and slides on the frictionless track shown in Figure. Determine
(a) the particle's speed at points B and C and
(b) the net work done by the gravitational force in moving the particle from A to C.