Simple Pendulum

General Physics Laboratory (PHY119) Fall 2014

1 Introduction

A pendulum in its simplest form consists of a weight suspended by a string pivoted at its upper end. Pendulums were used as a method for timekeeping in the past. This is due to their periodic motion. When the weight on the pendulum is displaced horizontally and then released, it will swing back and forth, in a periodic manner very close to a simple harmonic motion. In this experiment, the motion of the pendulum is investigated in order to calculate the gravitational acceleration.

2 Objective

1. Determine the value of the gravitational acceleration by using a simple pendulum.

3 Theory

A simple pendulum is any system consisting of a string with negligible mass having a length L pivoted at its upper end with a point mass (m) attached to the other end of the string.

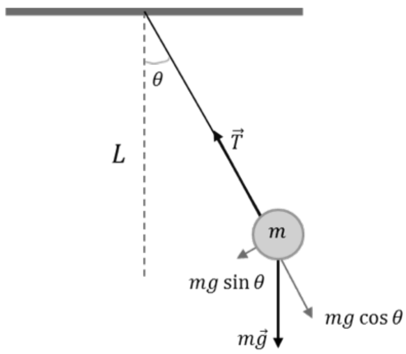

Figure 1: Simple pendulum

When the simple pendulum is allowed to swing, there are two forces acting on the point mass: the gravitational force mg and the force of tension T from the string. If the origin of the coordinate system is chosen to be at the center of the spherical mass in its equilibrium position, then we could resolve the two forces into a vertical and tangential component as seen in Figure 1, where both the tangential and vertical component of the gravitational force point in the negative direction. From Newton's second law on the tangential direction:

$$-mg\,\sin\theta = ma_t \ , \tag{1}$$

We can write the acceleration as the second derivative of the tangential position s which represents the arc opposite to the angle θ,

$$-g\ \sin\theta = \frac{d^2 s}{dt^2} \tag{2}$$

Since $s = L\theta$, we can write the tangential acceleration as,

$$-g\ \sin\theta = L\frac{d^2\theta}{dt^2} \tag{3}$$

In order to solve this equation, we limit our motion to small angles only. This way, we can use small angle approximation ($\sin\theta \approx \theta$),

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \tag{4}$$

This is a second order differential equation which has a solution on the form,

$$\theta = \theta_{max} cos(\omega t + \phi)\ , \tag{5}$$

where θ_{max} is the maximum angle position that a pendulum can reach, and ω is the angular frequency defined as,

$$\omega = \sqrt{\frac{g}{L}} \tag{6}$$

The period of motion is related to the angular frequency through the expression $T = 2\pi/\omega$. Thus, we can write the period as,

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{7}$$

Solving for the gravitational acceleration,

$$g = 4\pi^2\frac{L}{T^2} \tag{8}$$

This expression can be used to calculate the gravitational acceleration.

4 Equipment

- Simple pendulum "consist of spherical mass m hanging from a string of length l" .
- Stop watch .
- Metric ruler.

5 Procedure

1. Measure the radius of spherical mass r.
2. Calculate the length of the pendulum L from the expression $L = l + r$ where l is the length of the string(convert the unit from cm to m).

3. Displace the pendulum with small angle θ then release it.
4. Measure the time required to complete 10 cycle using your stopwatch.
5. Repeat steps 3 to 4 three times .
6. Calculate the average of t_1, t_2 and t_3 for 10 cycle .
7. Find the period (T).
8. Square the period (T^2).
9. Increase the string length by 5 cm .
10. Repeat steps 3 to 7 .
11. Increase the string length by 5 cm.
12. Repeat steps 3 to 7 .
13. Tabulate your data.

l (cm)	$L = l + r$ (m)	t_1 (s)	t_2 (s)	t_3 (s)	t_{avg} (s)	T (s)	T^2 (s^2)

14. Plot a graph between T^2 and L.
15. Draw the best line and find its slope.
16. Calculate the gravitational acceleration from the slope using equation (8) .
17. Find the percentage error.