Chapter 3
Capacitance and Dielectrics

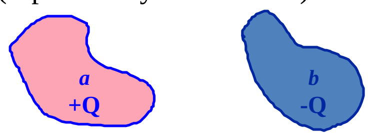
Chapter 3 **: Capacitance and Dielectrics**

Chapter 3 Outline:

Chapter 3. Capacitance and Dielectrics
Chapter 26 of the text book

26.1 Definition of Capacitance

Capacitor: two conductors (separated by an insulator) usually oppositely charged



The capacitance, C, of a capacitor is defined as a ratio of the magnitude of a charge on either conductor to the magnitude of the potential difference between the conductors

$$C = \frac{Q}{\Delta V}$$

Note that by definition capacitance is always a positive quantity. Because positive and negative charges are separated in the system of two conductors in a capacitor, there is electric potential energy stored in the system.

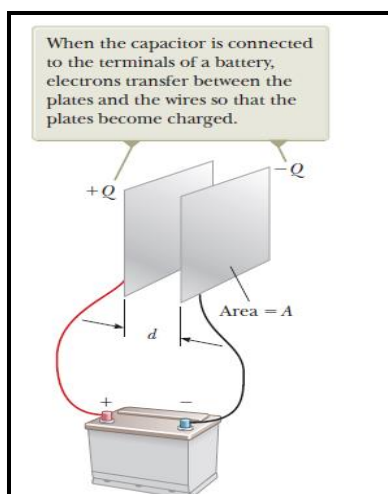
26.1 Definition of Capacitance

The unit of C is the farad (F), but most capacitors have values of C ranging from picofarads to microfarads (pF to μF).

$$1\,F = 1\,C/V$$

Recall, micro $\Rightarrow 10^{-6}$, nano $\Rightarrow 10^{-9}$, pico $\Rightarrow 10^{-12}$

If the external potential is disconnected, charges remain on the plates, so capacitors are good for storing charge (and energy).



26.2 Calculating Capacitance

Parallel-Plate Capacitors:

Two parallel metallic plates of equal area A are separated by a distance d. One plate carries a charge Q , and the other carries a charge -Q .

The surface charge density on each plate is σ=Q/A. If the plates are very close together (in comparison with their length and width), we can assume the electric field is uniform between the plates and zero elsewhere. The value of the electric field between the plates is

$$\boxed{E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}}$$

26.2 Calculating Capacitance

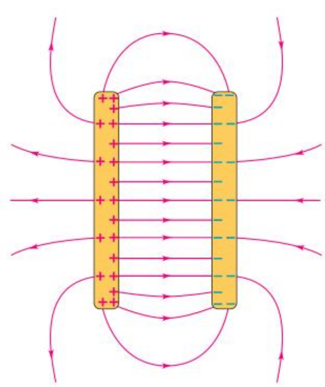
Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals Ed; therefore,

$$\boxed{\Delta V = Ed = \frac{Qd}{\epsilon_0 A}}$$

Substituting this result into C=Q/ΔV, we find that the capacitance is

$$\boxed{C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}}$$

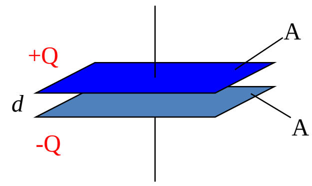
$$\boxed{C = \frac{\epsilon_0 A}{d}}$$



26.2 Calculating Capacitance

The capacitance of a device depends on the geometric arrangement of the conductors

$$C = \varepsilon_0 \frac{A}{d}$$



where A is the area of one of the plates, d is the separation, ε_0 is a constant called the permittivity of free space,

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N{\cdot}m^2$

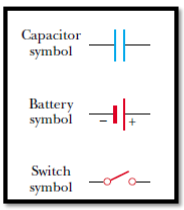
26.2 Calculating Capacitance

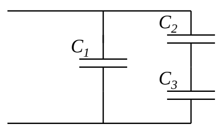
Example 26.1:

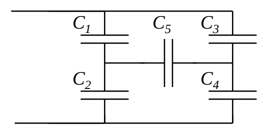
A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ m^2$ and a plate separation d = 1.00 mm . Find its capacitance?

26.3 Combinations of Capacitors

- Capacitors in circuits symbols
- It is very often that more than one capacitor is used in an electric circuit
- We would have to learn how to compute the equivalent capacitance of certain combinations of capacitors







26.3 Combinations of Capacitors

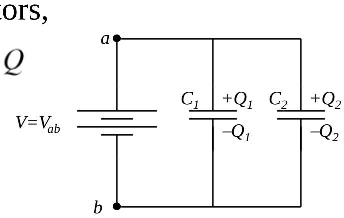
a. Parallel combination

Connecting a battery to the parallel combination of capacitors is equivalent to introducing the same potential difference for both capacitors,

$$V_1 = V_2 = V$$

A total charge transferred to the system from the battery is the sum of charges of the two capacitors,

$$Q_1 + Q_2 = Q$$



26.3 Combinations of Capacitors

Substituting these three relationships for charge into the Equation for charge ,we have

$$C_{eq}\,\Delta V = C_1\Delta V + C_2\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$

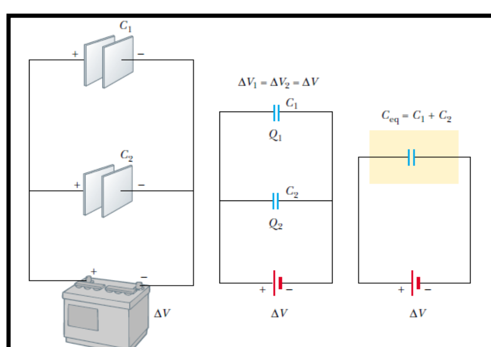
If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \ldots\ldots\ldots$$

It follows that the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors

26.3 Combinations of Capacitors

A 3 μF capacitor and a 6 μF capacitor are connected in parallel across an 18 V battery. Determine the equivalent capacitance and total charge deposited.



26.3 Combinations of Capacitors

b. Series combination

Connecting a battery to the serial combination of capacitors is equivalent to introducing the same charge for both capacitors,

$$Q_1 = Q_2 = Q$$

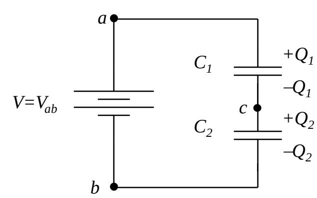
A voltage induced in the system from the battery is the sum of potential differences across the individual capacitors,

$$V = V_1 + V_2$$

$$Q_1 = C_1 V_1 \quad Q_2 = C_2 V_2$$

$$\frac{1}{C_{eq}} \equiv \frac{V}{Q} = \frac{V_1 + V_2}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{V_1}{Q_1} + \frac{V_2}{Q_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad Q_1 = Q_2 = Q \quad V_1 + V_2 = V$$



26.3 Combinations of Capacitors

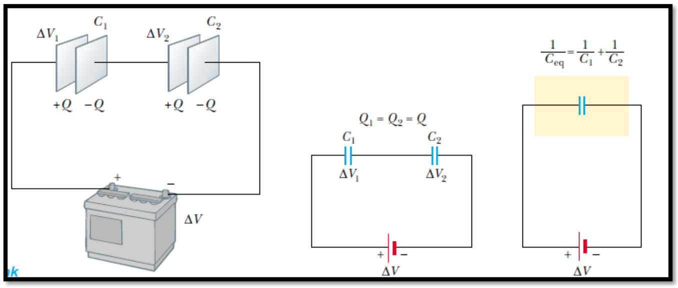
Analogous formula is true for any number of capacitors,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \quad \text{(series combination)}$$

It follows that the equivalent capacitance of a series combination of capacitors is always less than any of the individual capacitance in the combination

26.3 Combinations of Capacitors

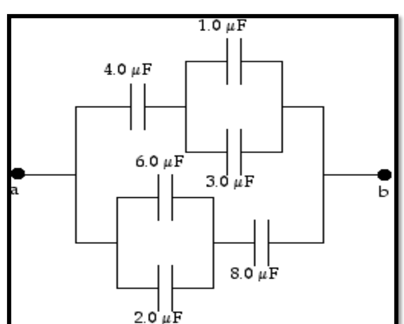
A 3 μF capacitor and a 6 μF capacitor are connected in series across an 18 V battery. Determine the equivalent capacitance.



26.3 Combinations of Capacitors

Example 26.4 Equivalent Capacitance

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.



26.4 Energy stored in a charged capacitor

We know that the work necessary to transfer an increment of charge dq from the plate carrying charge –q to the plate carrying charge +q (which is at the higher electric potential) is:

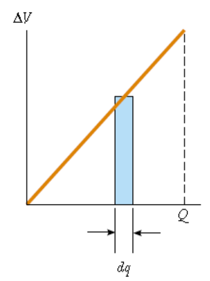
$$dW = \Delta V dq = \frac{q}{C} dq$$

So the total work, W, required to charge the capacitor from q=0 to the final charge q=Q is given by the integration:

$$W = \int_{q=0}^{q=Q} \frac{q}{C} dq$$

$$= \frac{1}{C}\int_{0}^{Q} q dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\therefore W = \frac{Q^2}{2C}$$



26.4 Energy stored in a charged capacitor

Note that the work done in charging the capacitor, W, is the same as electric potential energy U stored in the capacitor. Using the last equation, we can express the potential energy, U, stored in a charged capacitor in the following forms:

$$U = \frac{1}{2}QV = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

This result applies to any capacitor, regardless of its geometry. In practice, of course, there is a limit to the maximum energy (or charge) that can be stored. This is because at a sufficiently great value of ΔV, discharge ultimately occurs between the plates For this reason, capacitors are usually labeled with a maximum operating voltage

26.4 Energy stored in a charged capacitor

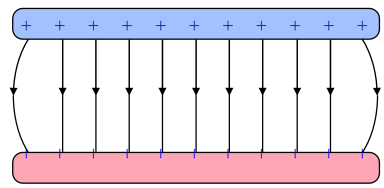
Find electric field energy density (energy per unit volume) in a parallel-plate capacitor

$$U = \frac{1}{2}CV^2$$

$$C = \frac{\varepsilon_0 A}{d} \quad volume = Ad \quad V = Ed$$

$$u \equiv U / volume = \text{energy density}$$

$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2 /(Ad)$$



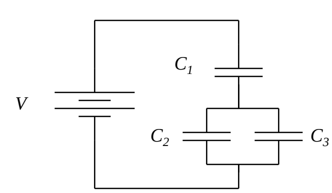
$$u = \frac{1}{2}\varepsilon_0 E^2$$

26.4 Energy stored in a charged capacitor

In the circuit shown V = 48V, C1 = 9mF, C2 = 4mF and C3 = 8mF.

(a) determine the equivalent capacitance of the circuit,

(b) determine the energy stored in the combination by calculating the energy stored in the equivalent capacitance.



26.5 Capacitors with Dielectrics:

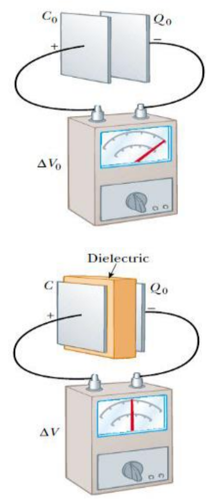
A dielectric is a "nonconducting" material such as rubber or glass. When a dielectric is inserted between the plates of a capacitor, the capacitance, C, increases. This increase is represented by a dimensionless quantity, $k \geq 1$, called the "dielectric constant" which is different for different materials

As shown in the figure, the charge on the plates remains unchanged after insertion of the dielectric material, but the potential difference decreases from ΔV_0 to $\Delta V = \Delta V_0/k$ (because EF strength decreases). Thus, the capacitance increases from C_0 to kC_0

In other words:

$$C = \frac{Q_o}{\Delta V} = \frac{Q_0}{\Delta V_0/_k} = k\frac{Q_0}{\Delta V_0}$$

$$C = kC_0$$



26.5 Capacitors with Dielectrics:

For a parallel-plate capacitor, where $C_0 = \varepsilon_0 A/d$, we can express the capacitance when the capacitor is filled with a dielectric as

$$C = k\frac{\varepsilon_o A}{d}$$

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

26.5 Capacitors with Dielectrics:

Example 26.6 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

(A) Find its capacitance.

(B) What is the maximum charge that can be placed on the capacitor?

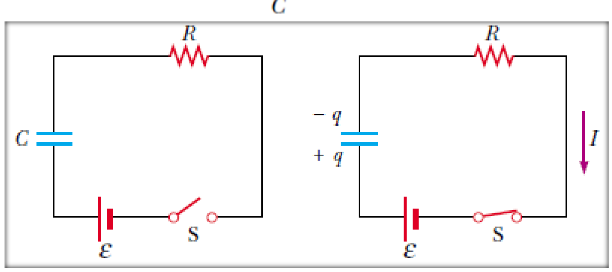
28.4 RC Circuits

A circuit containing a series combination of a resistor and a capacitor is called an RC circuit.

Charging a Capacitor:

- When the switch is closed at t = 0, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge.
- As the plates are being charged, the potential difference across the capacitor increases until it matches that supplied by the battery and the current stops.
- Applying Kirchhoff's loop rule to the circuit, traversing the loop clockwise gives:

$$\boldsymbol{\varepsilon} - \frac{q}{C} - IR = 0$$



28.4 RC Circuits

Charging a Capacitor:

- When the circuit is completed, the capacitor starts to charge.
- At the instant the switch is closed, the charge on the capacitor is zero.
- $I_0 = \frac{\varepsilon}{R}$ $(current\ at\ t = 0)$
- As the plates are being charged, the potential difference across the capacitor increases.
- Later, the capacitor continues to charge until it reaches its maximum charge (Q = Cε). And potential difference across the capacitor matches that supplied by the battery.
- Once the capacitor is fully charged, the current in the circuit is zero. Substituting I = 0 into Equation on previous slide gives the charge on the capacitor at this time:

The charge on the capacitor varies with time

$q(t) = C\varepsilon(1 - e^{-t/RC})$

$= Q(1 - e^{-t/RC})$

$Q = C\boldsymbol{\varepsilon}$ (maximum charge)

The current can be found

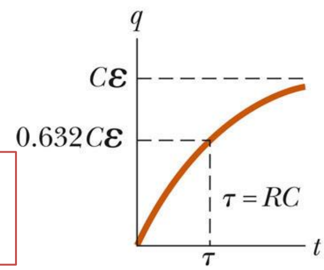
$I(t) = \frac{\varepsilon}{R} e^{-t/RC}$

τ is the time constant

$\tau = RC$

The energy stored in the charged capacitor is

½ $Q\mathcal{E}$ = ½ $C\mathcal{E}^2$.



28.4 RC Circuits

Discharging a Capacitor:

At some time t during the discharge, the current in the circuit is I and the charge on the capacitor is q . The loop equation for the circuit in Figure:

$$-\frac{q}{C} - IR = 0$$

Charge as a function of time for a discharging capacitor

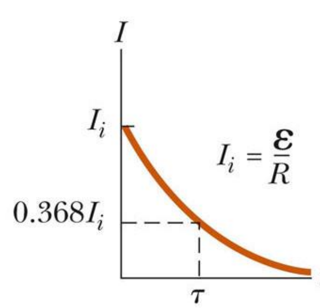
$$q(t) = Qe^{-t/RC}$$

Differentiating this expression with respect to time gives the instantaneous current as a function of time :

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right) = -\frac{Q}{RC}e^{-t/RC}$$

where $Q/RC = I_0$ is the initial current .

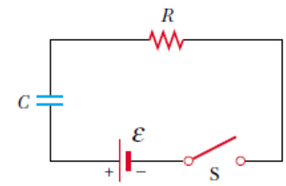
The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged..



28.4 RC Circuits

Example 28.12 Charging a Capacitor in an *RC* Circuit

An uncharged capacitor and a resistor are connected in series to a battery, as shown in Figure 28.23. If $\mathcal{E} = 12.0$ V, $C = 5.00\ \mu$F, and $R = 8.00 \times 10^5\ \Omega$, find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.



28.4 RC Circuits

Example 28.13 Discharging a Capacitor in an *RC* Circuit

Consider a capacitor of capacitance C that is being discharged through a resistor of resistance R, as shown in Figure 28.21.

(A) After how many time constants is the charge on the capacitor one-fourth its initial value?

(B) The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth its initial value?

28.4 RC Circuits

Example 28.14 Energy Delivered to a Resistor

A 5.00-μF capacitor is charged to a potential difference of 800 V and then discharged through a 25.0-kV resistor. How much energy is delivered to the resistor in the time interval required to fully discharge the capacitor?