# General Physics
# (Phys 106)



*Chapter 1 :* **Sources of the Magnetic Field**

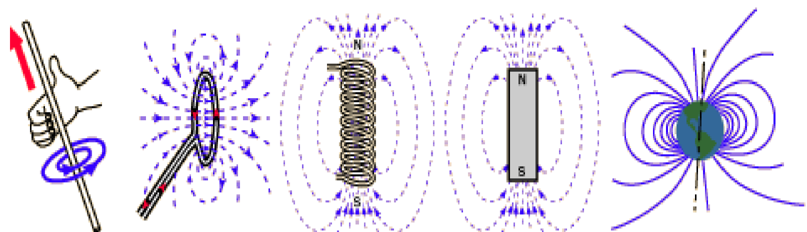
## Chapter 1 Outline:

**Chapter 1 Sources of the Magnetic Field**
**Chapter 30 in the text book**

1.1 The Biot-Savart`s law
1.2 The Magnetic Force Between Two Parallel Conductors
1.3 Amplere`s law
1.4 The magnetic Field of solenoid
1.5 Magnetic flux
1.6 Gauss`s low in magnetism
1.7 Displacement current and generalized Ampere`s law

# Magnetic Field Sources

Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.




Magnetic Field Sources

# 1.1 Biot-Savart Law

**Biot and Savart** performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, they arrived at a mathematical expression that gives the magnetic field dB at some point in space P at a distance r in terms of the current I in a segment of the conductor a length element ds that produces the field:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(d\vec{s}\times\vec{r}\right)}{r^2}$$

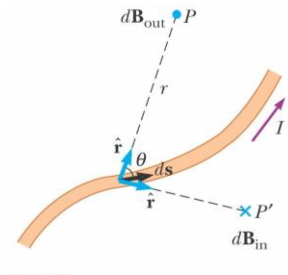
This expression know today as the Biot-Savart Law

where $\mu_0$ is a constant called the permeability of free space

$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T.m/A}.$$

# 1.1 Biot-Savart Law

So , , , Biot and Savart produced an equation that gives the magnetic field at some point in space in terms of the current that produces the field.




To find the total ***B*** created by a finite size current-carrying conductor at some point in space, we must integrate the previous equation: M

$$\int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{(d\vec{s} \times \vec{r})}{r^2}$$

# 1.1 Biot-Savart Law

**Quick Quiz 30.1** Consider the current in the length of wire shown in Figure 30.2. Rank the points *A*, *B*, and *C*, in terms of magnitude of the magnetic field due to the current in the length element shown, from greatest to least.

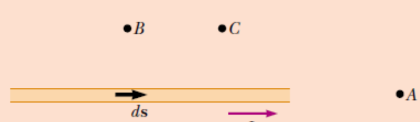
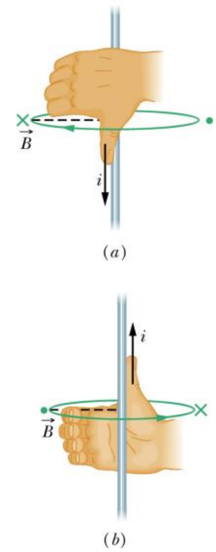


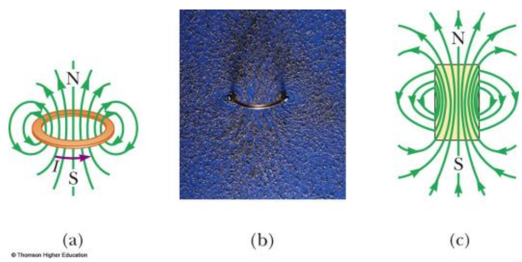

**Figure 30.2** (Quick Quiz 30.1) Where is the magnetic field the greatest?

# Right-Hand Rule

The right hand rule can be used to determine the direction of the magnetic field around the current carrying conductor:

- Thumb of the right hand in the direction of the current.
- Fingers of the right hand curl around the wire in the direction of the magnetic field at that point.

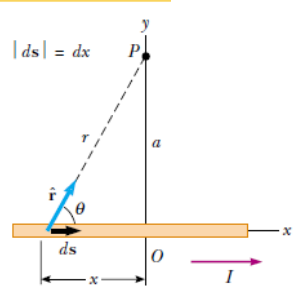



# 1.1 Biot-Savart Law

**Example 30.1 Magnetic Field Surrounding a Thin, Straight Conductor**

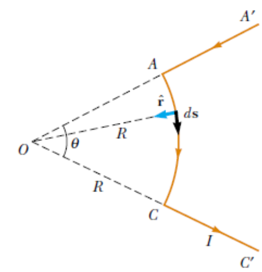
Consider a thin, straight wire carrying a constant current $I$ and placed along the $x$ axis as shown in Figure 30.3. Determine the magnitude and direction of the magnetic field at point $P$ due to this current.



# 1.1 Biot-Savart Law

**Example 30.2 Magnetic Field Due to a Curved Wire Segment**

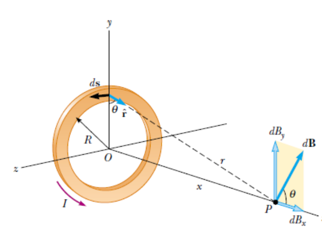
Calculate the magnetic field at point $O$ for the current-carrying wire segment shown in Figure 30.5. The wire consists of two straight portions and a circular arc of radius $R$, which subtends an angle $\theta$. The arrowheads on the wire indicate the direction of the current.




# 1.1 Biot-Savart Law

**Example 30.3 Magnetic Field on the Axis of a Circular Current Loop**

Consider a circular wire loop of radius $R$ located in the $yz$ plane and carrying a steady current $I$, as in Figure 30.6. Calculate the magnetic field at an axial point $P$ a distance $x$ from the center of the loop.

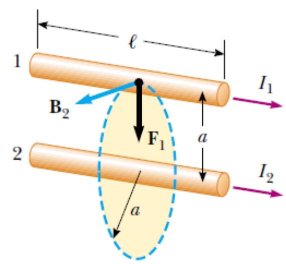


## 1.2 The Magnetic Force Between Two Parallel Conductors:

- Consider two long, straight, parallel wires separated by a distance a and carrying currents $I_1$ and $I_2$ in the same direction.
- Wire 2, which carries a current $I_2$ and is identified arbitrarily as the source wire, Creates a magnetic field $\mathbf{B}_2$ at the location of wire 1, the test wire. The direction of $\mathbf{B}_2$ is perpendicular to wire 1, as shown.
- The field $\mathbf{B}_2$ due to the current in wire 2 exerts a force on wire 1 of $\mathbf{F}_1 = I_1 \boldsymbol{l} \times \mathbf{B}_2$ Because $\boldsymbol{l}$ is perpendicular to $\mathbf{B}_2$ in this situation.

Substituting the equation for $\mathbf{B}_2$ gives

$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}\,\ell$$




## 1.2 The Magnetic Force Between Two Parallel Conductors:

The direction of $\mathbf{F}_1$ is toward wire 2 because $\boldsymbol{l} \times \mathbf{B}_2$ is in that direction. If the field set up at wire 2 by wire 1 is calculated, the force $\mathbf{F}_2$ acting on wire 2 is found to be equal in magnitude and opposite in direction to $\mathbf{F}_1$, according to Newton's third law

- ❑ Parallel conductors carrying currents in the same direction attract each other
- ❑ Parallel conductors carrying current in opposite directions repel each other

Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply $F_B$. We can rewrite this magnitude in terms of the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

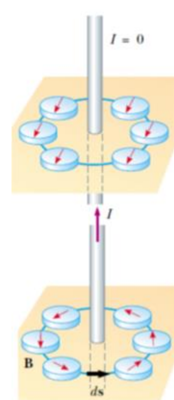
## 1.2 The Magnetic Force Between Two Parallel Conductors:

- The force between two parallel wires can be used to define the ampere
- When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is $2 \times 10^{-7}$ N/m, the current in each wire is defined to be 1 A

**Quick Quiz 30.2** For $I_1 = 2$ A and $I_2 = 6$ A in Figure 30.8, which is true: (a) $F_1 = 3F_2$, (b) $F_1 = F_2/3$, (c) $F_1 = F_2$?

## 1.3 Ampère's Law:

- Ampere's law is the equivalent of Gauss's law in electric fields. Instead of a closed surface we consider a closed loop called the Amperian loop.
- Let us demonstrate a simple experiment that shows how the a current-carrying conductor produces magnetic fields. Several compass needles are placed in a horizontal plane near a long vertical wire.
- When no current is present in the wire, all the needles point in the same direction (that of the Earth magnetic field), as expected. When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle.
- When the current is reversed, the needles also reverse.



# 1.3 Ampère's Law:

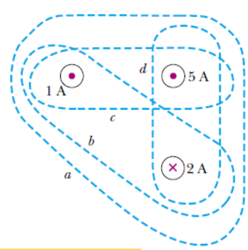
Now let us evaluate the product **B**.ds for a small length element ds on the circular path, and sum the products for all elements over the closed circular path. Along this path, the vectors d**s** and **B** are parallel at each point, so **B**.ds = B ds.

Furthermore, the magnitude of B is constant on this circle. Therefore, the sum of the products B ds over the closed path, which is equivalent to the line integral of **B**.ds, is:

$$\oint \vec{B}.d\vec{s} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

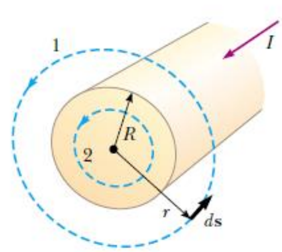
# 1.3 Ampère's Law:

**Quick Quiz 30.4** Rank the magnitudes of $\oint \mathbf{B}\cdot d\mathbf{s}$ for the closed paths in Figure 30.10, from least to greatest.




**Example 30.4 The Magnetic Field Created by a Long Current-Carrying Wire**

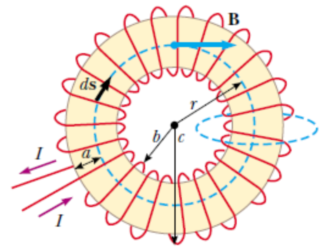
A long, straight wire of radius $R$ carries a steady current $I$ that is uniformly distributed through the cross section of the wire (Fig. 30.12). Calculate the magnetic field a distance $r$ from the center of the wire in the regions $r \geq R$ and $r < R$.



# 1.3 Ampère's Law:

**Example 30.5 The Magnetic Field Created by a Toroid**

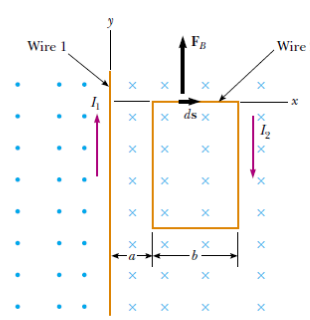
A device called a *toroid* (Fig. 30.14) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material. For a toroid having $N$ closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance $r$ from the center.




# 1.3 Ampère's Law:

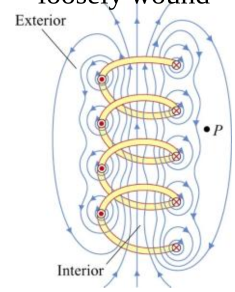
**Example 30.7 The Magnetic Force on a Current Segment**

Wire 1 in Figure 30.16 is oriented along the $y$ axis and carries a steady current $I_1$. A rectangular loop located to the right of the wire and in the $xy$ plane carries a current $I_2$. Find the magnetic force exerted by wire 1 on the top wire of length $b$ in the loop, labeled "Wire 2" in the figure.



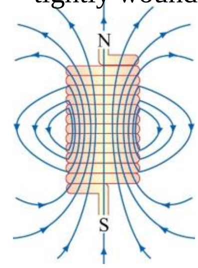
## 1.4 The Magnetic Field of a Solenoid:

loosely wound




- A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire which call the interior of the solenoid
- The field lines in the interior are
  - approximately parallel to each other
  - uniformly distributed
  - close together
- The field is strong and almost uniform in the interior
- The magnetic field at exterior points such as P is weak because the magnetic field due to current elements on the right-hand portion of a turn tends to cancel the magnetic field due to current elements on the left-hand portion.

tightly wound




### Tightly Wound Solenoid

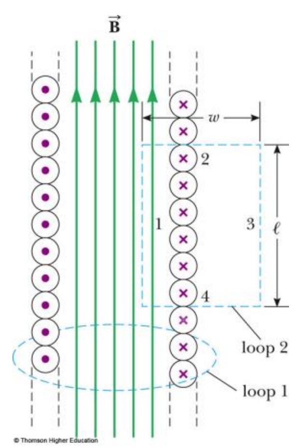
- The field distribution is similar to that of a bar magnet
- As the length of the solenoid increases
  - the interior field becomes more uniform
  - the exterior field becomes weaker

## 1.4 The Magnetic Field of a Solenoid:

- An ideal solenoid is approached when:
  - the turns are closely spaced
  - the length is much greater than the radius of the turns
- B in the interior space is uniform and parallel to the axis, and B in the exterior space is zero
- Consider an ideal solenoid of total length l and number of turns N.
- Ampere’s law can be used to find the interior magnetic field of the solenoid
- Consider a rectangle with side ℓ parallel to the interior field and side w perpendicular to the field

(This is loop 2 in the diagram)

The side of length ℓ inside the solenoid contributes to the field (This is side 1 in the diagram)



# 1.4 The Magnetic Field of a Solenoid:

- Applying Ampere's Law gives

$$\oint \vec{B}\cdot d\vec{s} = \int_{path1} \vec{B}\cdot d\vec{s} = B\int_{path1} ds = Bl$$

- The total current through the rectangular path equals the current through each turn multiplied by the number of turns

$$\oint \vec{B}\cdot d\vec{s} = Bl = \mu_o NI$$

- Solving Ampere's law for the magnetic field is

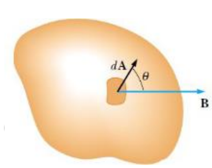
$$B = \mu_o \frac{N}{\ell} I = \mu_o n I$$

n = N / ℓ is the number of turns per unit length

# 1.5 Magnetic Flux

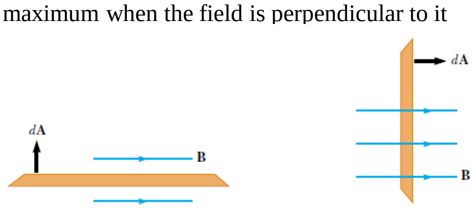
Consider an element of area dA. If the magnetic field at this element is $\boldsymbol{B}$, the magnetic flux through the element is $\boldsymbol{B}.d\boldsymbol{A}$, where $d\boldsymbol{A}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area $d\boldsymbol{A}$. Therefore, the total magnetic flux $\Phi_B$ through the surface is

$$\phi_B = \int \vec{B}.d\vec{A}$$




The unit of magnetic flux is T. $m^2$, which is defined as a Weber

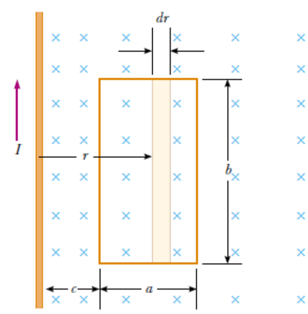
- The flux is zero when the field is parallel to the surface
- The flux is a maximum when the field is perpendicular to it



# 1.5 Magnetic Flux

**Example 30.8 Magnetic Flux Through a Rectangular Loop**

A rectangular loop of width $a$ and length $b$ is located near a long wire carrying a current $I$ (Fig. 30.22). The distance between the wire and the closest side of the loop is $c$. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

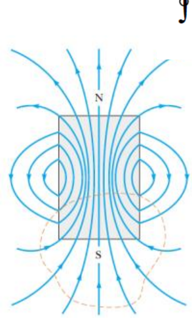


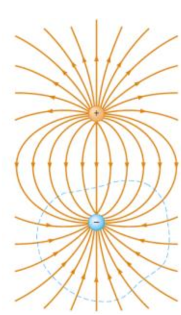

# 1.6 Gauss's Law in Magnetism:

Note that for any closed surface, such as the one outlined by the dashed line in Figure, the number of lines entering the surface equals the number leaving the surface; thus, the net magnetic flux is zero.

The net magnetic flux through any closed surface is always zero.

$$\oint B.dA = 0$$



## 1.7 Displacement Current and the General Form of Ampère's Law:

Consider a capacitor that is being charged as illustrated in Figure. When a conduction current is present, the charge on the positive plate changes but no conduction current exists in the gap between the plates. Now consider the two surfaces S1 and S2 in Figure, bounded by the same path P. Ampère's law states that $\oint \boldsymbol{B}.d\boldsymbol{s}$ around this path must equal $\mu_0 I$, where $I$ is the total current through any surface bounded by the path P.

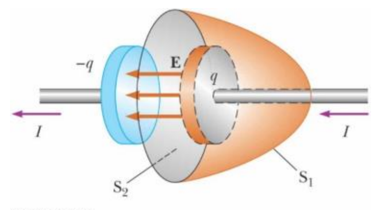
When the path P is considered as bounding S1, $\oint \boldsymbol{B}.d\boldsymbol{s} = \mu_0 I$ because the conduction current passes through S1. When the path is considered as bounding S2, however, $\oint \boldsymbol{B}.d\boldsymbol{s} = 0$ because no conduction current passes through S2. Thus, we have a contradictory situation that arises from the discontinuity of the current!

This current is called the displacement current and is given by:

$$I_d = \varepsilon_o \frac{d\phi_E}{dt}$$

Where ϵo is the permittivity of the free space.

and is the $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$ electric flux




## 1.7 Displacement Current and the General Form of Ampère's Law:

If the current pass through the conductor is not constant, another current component is generated due to the change in magnetic flux at the conductor by induction. Maxwell modified Ampere's law to include time-varying electric fields. The new "general" law states that: "magnetic fields are produced both by conduction currents and by time-varying electric fields"

With $I_d$, we can express the general form of Ampère's law (sometimes called the Ampère–Maxwell law) as

$$\oint \vec{B}.d\vec{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

If the field is constant in time, then $I_d$=0 and we get Ampere's law

## 1.7 Displacement Current and the General Form of Ampère's Law:

**Example 30.9 Displacement Current in a Capacitor**

A sinusoidally varying voltage is applied across an 8.00-$\mu$F capacitor. The frequency of the voltage is 3.00 kHz, and the voltage amplitude is 30.0 V. Find the displacement current in the capacitor.