## Midterm 2 (24/11/2015)

## Answer the following questions

## Question 1. $(8=5+3)$ marks

a) Find the first derivative of each the following functions:

$$
\begin{aligned}
& {[1] y=4^{x^{2}+x+1}+e^{\sin x},} \\
& {[2] y=\tan ^{-1}\left(x^{3}\right)-4 \cos ^{-1}\left(x^{2}\right),} \\
& {[3] y=\ln \left(x^{2} \sec x\right) .}
\end{aligned}
$$

b) Find the value of the constant $c$ which satisfies the mean value theorem for the function $f(x)=x^{4}-4 x+10$ on the closed interval $[-1,1]$.

Question 2. $(7=4+3)$ marks
a) Evaluate the following limits:
(i) $\lim _{x \rightarrow 1} \frac{\ln x}{x-1}$
(ii) $\lim _{x \rightarrow \infty} \frac{e^{x}+\ln 4 x}{x^{2}}$
b) Use the implicit differentiation to determine the equation of the tangent line of the function $y$, defined by the equation $x^{2}+y^{3}-2 y=8$ at the point $(3,1)$.

Question 3. $(5=2.5+2.5)$ marks
a) Prove that 2 and -1 are the critical points of the function $f(x)=2 x^{3}-3 x^{2}-12 x$.
b) Deduce the values of absolute extrema (minimum and maximum) of $f(x)$ on the closed interval $[-2,4]$.

