KINGDOM OF SAUDI ARABIA

Ministry of Education

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College of Science

Department: Mathematics \&
Statistics

1435/1436H 2nd Semester
Course Code \&Number: MAT 106
Course Title: ENG/Calculus 2
Date of Exam: 17-05-2015

Duration: 2.5 Hours

## Final Examination

## Question 1. $(2+2+3+2$ points $):$

Evaluate the following integrals
i) $\int \frac{\sqrt{x^{2}-4}}{x} d x$
ii) $\int \cos ^{3} x \sin ^{3} x d x$
iii) $\int_{-\infty}^{0} \frac{e^{x}}{1+e^{x}} d x$,
iv) $\int \frac{\ln x}{x^{3}} d x$

Question 2. $(3+3+2+4$ points $)$ :
(1)Investigate the convergence or divergence of the following series:
i) $\sum_{k=3}^{\infty} \frac{1}{k(\ln k)^{2}}$,
ii) $\sum_{k=0}^{\infty}(-1)^{k}\left(\frac{2 k+1}{3 k+1}\right)^{k}$,
iii) $\sum_{k=1}^{\infty} \frac{k^{2}+10 k+20}{k^{3}+10 k+10}$.
(2) Determine the interval and radius of convergence for the power series

$$
\sum_{k=1}^{\infty} \frac{(x+1)^{k}}{k^{2} 2^{k}}
$$

## Question 3. (3+4+4points ):

(1) Show that the following limit does not exist: $\lim _{(x, y) \rightarrow(0,0)} \frac{2 x^{2} y}{x^{4}+y^{2}}$
(2) For $f(x, y)=\cos (x y)-x^{3} y^{4}$, show that $f_{x y}=f_{y x}$.
(3) Compute the directional derivative $D_{u} f(3,-4)$ for the function $f(x, y)=\sqrt{x^{2}+y^{2}}$ in the direction of the vector $u=\langle 3,-2\rangle$.

## Question 4. (4+4 points ):

(1) Evaluate $\iint_{R} 2 x y d A$, where R is the region bounded by the graphs of $y=x^{2}$ and $x+y=2$.
(2) Evaluate the following integral by changing the order of integration: $\int_{0}^{1} \int_{y^{2}}^{1} 2 y e^{x^{2}} d x d y$.

Good Luck

