Chapter 3

Rates of return

3.1 Cash-flows

Any investment project has multiple cash-flows at different time.

(say $C_0, C_1, \ldots \quad , C_n$ at time t_0 , t_1 , …, t_n).

These cash-flows can be positive (representing returns) or negative (representing costs or losses)

3.2 Net Present Value (NPV)

Using an effective interest rate i, the net present value of an investment project with n cash-flows is the sum of the discounted cash-flows, namely

$$NPV(i) = \sum_{k} \frac{C_k}{(1+i)^k}$$
$$= C_0 + \frac{C_1}{(1+i)^1} + \cdots + \frac{C_n}{(1+i)^n}$$

Example

A project requires an initial of \$1000 now, and it will return \$800 in 2 years and \$1100 in 5 years. Compute NPV(0.05) ?

Example

A project requires an initial of $500 now and $700 in 3 years, and will return $1000 in 2 years and $1200 in 4 years.

Compute NPV(0.08) ?

3.3 Internal rate of return (IRR)

The IRR for a project is the annual effective rate of interest in such that

$$NPV\ (i)=0$$

Example

A project requires an initial of \$1000 now, and it will return \$800 in 2 years and \$1200 in 4 years. Find the IRR.

3.4 Investment Fund:

An investment fund usually is a firm that invests the pooled funds of investors for a fee. In this section we examine a method of determining the interest rate earned by an investment fund.

In practice, it is common for a fund to be incremented with new principal deposits, decremented with principal withdrawals, and incremented with interest earnings many times throughout a period.

We have:

A = the amount in the fund at the beginning of the period, i.e. $t = 0$.

B = the amount in the fund at the end of the period, i.e. $t = 1$.

I = the amount of interest earned during the period.

c_t = the net amount of principal contributed at time t (inflow less outflow at time t) where $0<t<1$.

C = total net amount of principal contributed during the period.

3.4.1 Dollar Weighted Rate of Return (DWRR)

The DWRR is the return rate earned by an investment fund during a period (one year in general.

By a simple interest approximation, we obtain

$$DWRR = \frac{B - A - C}{A + \sum_t C_t(1 - t)}$$

Example

The fund value at the beginning of the year is $10000 and at the end of the 3rd month, $2000 was invested (added),and at the end of the 7th month, $1500 was withdrawn. At the end of the year, the fund value is $12000. Find DWRR?

3.4.2 Time Weighted Rate of Return (TWRR)

The dollar-weighted rate of interest depends on the precise timing and amount of the cash-flows. In practice, professional fund managers who direct investment funds have no control over the timing or amounts of the external cash flows. Therefore, if we are comparing the performance of different fund managers, the dollar weighted rate of interest does not always provide a fair comparison.

In this section, we consider an alternative measure of performance that does not depend on the size or the timing of the cash flows, namely the time-weighted rate of interest also known as the time-weighted rate of return.

Suppose m deposits or withdrawals are made during a year at times t_1, t_2, …, t_m (so that no contributions at $t_0=0$ and $t_{m+1}= 1$). Thus, the year is divided into m+1 subintervals. For k=1, 2,…, m+1 we let j_k be the yield (return) rate over the k^{th} subinterval.

For k=1,…,m let C_{t_k} be the net contribution at exact time t_k and B_{t_k} the value of the fund before the contribution at time t_k: Note that $C_0 = C_{m+1} = 0$ and B_0 is the initial investment and B_1 is the value of the fund at the end of the year.

The yield rate of the fund from time t_{k-1} to time t_k satisfies the equation of value

$$B_{t_k} = 1 + j_k\left(B_{t_{k-1}} + C_{t_{k-1}}\right)$$

or

$$1 + j_k = \frac{B_{t_k}}{B_{t_{k-1}} + C_{t_{k-1}}}$$

$k=1,2,\ldots,m+1$.

The overall yield rate i for the entire year is given by $1+TWRR = (1+j_1)(1+j_2)\ldots(1+j_{m+1})$

Or $TWRR = (1+j_1)(1+j_2)\ldots(1+j_{m+1}) - 1$

Example

At the start of the year, the fund value is \$10000 and at the end of the 3rd month, the fund value is \$11000 and \$2000 was added. At the end of the 7th month, the fund value is \$10000 and \$1500 was withdrawn. The fund value at the end of the year is \$12000. Compute TWRR?

Example

At January 1st, the fund value is \$12000 . and at April 1st the fund value \$13000 and \$2000 was withdrawn.

At August 1st the fund is \$10000 and \$3000 was added and at the end of the year the fund value is \$14000.

Compute DWRR and TWRR?