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Course	**Financial Mathematics**
Unit course	**FIN 118**
Number Unit	**7**
Unit Subject	**Sequences & Series**

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we will see in this unit

✓The "arithmetic sequences" and "arithmetic series".

✓ The "Geometric sequences" and "Geometric series".

✓ Solve some questions for real world situations in order to solve problems, especially economic and financial.

Learning Outcomes

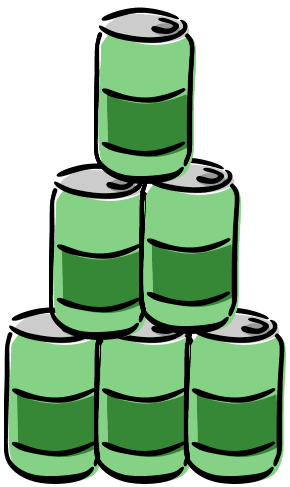
At the end of this chapter, you should be able to:

1. Understand what is meant by "Arithmetic sequences" and "Arithmetic series"

2. Understand what is meant by "Geometric sequences" and "Geometric series".

3. Solve some questions for real world situations in order to solve problems, especially economic and financial.

Arithmetic sequences and series



The can pyramid…

Q1/ How many cans are there on the bottom row in this pyramid ?

Q2/ How many cans are there in this pyramid ?

1/ There are 3 cans on the bottom row

2/ There are 6=3+2+1 cans in this pyramid

!!! How many cans are there in a pyramid with 100 cans on the bottom row?

Arithmetic sequences and series

Solution:

We have S = 100+99+98+….+3+2+1 cans in a pyramid with 100 cans on the bottom row.

$$S = 100+99+98+\dots+3+2+1$$
$$S = 1+2+3+\dots+98+99+100$$
$$2S = \underbrace{101+101+101+\dots+101+101+101}_{\text{100 times}}$$

Then

$$S = \frac{100\times 101}{2} = 5050$$

Arithmetic sequences

Definition1: An Arithmetic Sequence is a sequence whose consecutive terms have a common difference, r.

- In the pyramid can example the common difference r = 1.

Example 1:

0 2 4 6 8 10 12 14 16 18 ? ? ?

→ To find the common difference (r), just subtract any term from the term that follows it. Then

The next numbers are 20, 22 and 24 because the common difference is r = 2.

!!! Question:

What is the twentieth number ?

Arithmetic sequences

Example 1 (Continued)

If We set $u_1=0$, $u_2=2$, $u_3=4$, $u_4=6$, $u_5=8$, and so on. Then the twentieth number correspond to u_{20}

First Term: $u_1 = 0$

Second Term: $u_2 = u_1 + r = 0 + 2 = 2$

Third Term: $u_3 = u_2 + r = u_1 + 2r = 0 + 2\times 2 = 4$

Fourth Term: $u_4 = u_3 + r = u_1 + 3r = 0 + 3\times 2 = 6$

Fifth Term: $u_5 = u_4 + r = u_1 + 4r = 0 + 4\times 2 = 8$

And so on:

$$u_{20} = u_{19} + r = u_1 + 19r = 0 + 19\times 2 = 38$$

Formula for the n[th] term of an ARITHMETIC sequence

$$u_n = u_1 + (n-1)r$$

- n[th] term
- 1[st] term
- # of term trying to find
- common difference

Example 2:

Given the following arithmetic sequence:

100, 120, 140, 160,... Find the 10th term

$u_1 = 100 \quad r = 20 \quad u_{10} = u_1 + 9r = 100 + 9 \times 20 = 280$

Arithmetic sequences

Example3:

Which of the following sequences are arithmetic? Identify the common difference, and calculate u_{12} for arithmetic sequences.

$$-3, -1, 1, 3, 5, 7, 9, \ldots$$

$$84, 80, 74, 66, 56, 44, \ldots$$

$$15.5, 14, 12.5, 11, 9.5, 8, \ldots$$

$$-50, -44, -38, -32, -26, \ldots$$

Aerithmetic series

Definition:

An Arithmetic series is the sum of the terms in an arithmetic Sequence.

If we consider the following arithmetic sequence

$u_1, u_2, u_3, u_4, u_5, \ldots., u_n$ then $S_n = \sum_{i=1}^{n} u_i$

We can write the formula of arithmetic series as:

# of terms

$$S_n = \frac{n}{2}(u_1 + u_n)$$

1st Term

Last Term

Arithmetic series

Example 1:

Find the 20th term and the sum of 20 first terms of the sequence 2 , 5, 8, 11, 14, 17, . . .

Solution:

This is an arithmetic sequence with

- $u_1 = 2, \quad r = 3 \quad and \;\; for \;\; n = 20$ we have

$$\Rightarrow u_{20} = 2 + 19(3) = 59$$

$$S_n = \frac{n}{2}\left(u_1 + u_n\right) \qquad \Rightarrow S_{20} = \frac{20}{2}\left(u_1 + u_{20}\right)$$

$$\Rightarrow \; S_{20} = \frac{20}{2}\left(2 + 59\right) = 610$$

Arithmetic series

Example 2:

Find the sum of the terms of this arithmetic series.

$$151+147+143+139+.....+(-5)$$

Solution:

Geometric sequences and series

Example:

What if your interest check started at \$100 a week and doubled every week. What would your interest after three weeks? What would your interest after nine weeks? What would your total interest at the end of tenth week?

Solution:

* Starting \$100.
* After one week : $2 \times \$100 = \$200$
* After two weeks : $2 \times \$200 = \$400 = 2^2 \times 100$
* After three weeks : $2 \times \$400 = \$800 = 2^3 \times 100$

!! After 3 weeks I would have \$800

Geometric sequences and series

Solution: continued

- Starting $100.
- After one week : $2 \times \$100 = \$200 = 2^{1} \times 100$
- After two weeks : $2 \times \$200 = \$400 = 2^{2} \times 100$
- After three weeks : $2 \times \$400 = \$800 = 2^{3} \times 100$

And so on

- After nine weeks : $2^{9} \times 100 = \$51200$

!!! After 9 weeks I would have $51200

- The total interest is:

$$S = 100 + 200 + 400 + 800 + ... + 51200$$

$$S = 100\left(1 + 2 + 2^{2} + 2^{3} + ... + 2^{9}\right)$$

Geometric sequences and series

Solution: continued

$$S = 100\left(1 + 2 + 2^2 + 2^3 + \dots + 2^9\right)$$

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$$2S = 100\left(2 + 2^2 + 2^3 + 2^4 + \dots + 2^{10}\right)$$

$$S - 2S = 100\left(1 - 2^{10}\right)$$

$$\left(1 - 2\right)S = 100\left(1 - 2^{10}\right)$$

$$S = 100\frac{\left(1 - 2^{10}\right)}{\left(1 - 2\right)} = 102300$$

!!! At the end of tenth week I would have $102300

Geometric sequences

Definition1:

A Geometric Sequence is a sequence whose consecutive terms have a common ratio, q.

Example 1:

→ In interest example the common ratio $q = 2$.

suppose we have the following sequence

1 4 16 64 ? ?

→ To find the common ratio (q), just divide any term by the previous term. Then

The next numbers are 256 and 1024 because the common ratio is $q = 4$.

!!! **Question:**

What is the tenth number ?

Geometric sequences

If We set u_1=1, u_2=4, u_3=16, u_4=64, and so on.
Then the tenth number correspond to u_{10}

First Term: $u_1 = 1$

Second Term: $u_2 = u_1 \times q = 1 \times 4 = 4$

Third Term: $u_3 = u_2 \times q = u_1 \times q^2 = 1 \times 4^2 = 16$

Fourth Term: $u_4 = u_3 \times q = u_1 \times q^3 = 1 \times 4^3 = 64$

Fifth Term: $u_5 = u_4 \times q = u_1 \times q^4 = 1 \times 4^4 = 256$

And so on:

$$u_{10} = u_9 \times q = u_1 \times q^9 = 1 \times 4^9 = 262144$$

Formula for the nth term of a GEOMETRIC sequence

$$u_n = u_1 \times q^{n-1}$$

nth term

1st term

common ratio

Example 2:

Given the following geometric sequence:

5, 15, 45,… Find the 10th term.

Geometric sequences

Example3:

Which of the following sequences are geometric? Identify the common ratio, and calculate u_5 for geometric sequences:

* 2, 6,18,… .

* 5, 15, 25, 45, ….

* 1, 5, 25, 125, ….

Geometric series

Definition:

A Geometric Series is the sum of the terms in a geometric Sequence.

If we consider the following geometric sequence

$$u_1, u_2, u_3, u_4, u_5, \ldots., u_n \quad \text{then} \quad S_n = \sum_{i=1}^{n} u_i$$

We can write the formula of geometric series as:

$$S_n = u_1 \times \frac{(1-q^n)}{(1-q)}$$

- The sum of n terms → S_n
- 1st Term → u_1
- # of terms → n
- The common ratio → q

Geometric sequences and series

Example:

1/ Find the common ratio of the following sequence
2, -4, 8, -16, 32, …

2/ Find the ninth and tenth term.

3/ Find S_{10}.

Solution:

Arithmetic and geometric sequences and series

Time to Review!

Arithmetic sequences and series	Geometric sequences and series
$u_n - u_{n-1} = r$	$\frac{u_n}{u_{n-1}} = q$
$u_n = u_1 + (n-1) \times r$	$u_n = u_1 \times q^{n-1}$
$S_n = \frac{n(u_1 + u_n)}{2}$	$S_n = u_1 \times \left[\frac{1-q^n}{1-q}\right]$

That's All !

Practical Examples

Example 1: (Arithmetic sequence)

Abdul Aziz makes a monthly saving plan for a period of two years in a bank that doesn't give interest for saving accounts (compliant with Shariah). In the first month he deposits 1000 SAR, in the second month he deposits 1200 SAR and the third month 1400 SAR and so on.

1. What is the amount that he will deposit in the fifth month, in the 24th month?
2. What is the total amount that he will obtain at the end of the second year?

Practical Examples

Solution of Example1

1. The saving plan has the form of an arithmetic sequence with a first term u_1 = 1000 and common difference r = 200.

Use $u_n = u_1 + (n-1)\times r$

$u_n = 1000 + (n-1) \times 200$

$u_n = 800 + 200n$

Then $u_5 = 800 + 200\times 5 = 1800$ SAR

and $u_{24} = 800 + 200\times 24 = 5600$ SAR

2. Total amount obtained at the end of the second year: $S_{24} = u_1 + u_2 + \dots + u_{24}$

$$S_{24} = \frac{24}{2}\left(u_1 + u_{24}\right) = 12\times\left(1000 + 5600\right) = 79200\,\text{SAR}$$

Practical Examples

Example 2: (Geometric sequence)

Abdul Aziz makes a monthly saving plan for a period of one year in a bank that doesn't give interest for saving accounts (compliant with Shariah). In the first month he deposits 5 SAR, in the second month he deposits 10 SAR and the third month 20 SAR and so on.

1. What is the amount that he will deposit in the fourth month, in the eighth month.

2.What is the total amount that he will obtain at the end of the period.

Practical Examples

Solution of Example2

1. The saving plan has the form of a geometric sequence with a first term $u_1 = 5$ and common ratio $q = 2$.

Use $u_n = u_1 \times q^{n-1}$

$u_n = 5 \times 2^{n-1}$

Then $u_4 = 5 \times 2^{(4-1)} = 40$ SAR

and $u_8 = 5 \times 2^{(8-1)} = 640$ SAR

2. Total amount obtained at the end of the period:

$S_{12} = u_1 + u_2 + \ldots + u_{12}$

$$S_{12} = u_1 \times \left(\frac{1-q^{12}}{1-q}\right) = 5 \times \left(\frac{1-2^{12}}{1-2}\right) = 20475\,\text{SAR}$$

we will see in the next unit

- ✓ The relationship between time and money.
- ✓ The simple interest rate and the interest amount
- ✓ The present value of one future cash flow
- ✓ The future value of an amount borrowed or invested.
- ✓ The relationship between Real Interest Rate, Nominal Interest Rate and Inflation.