

Al-Imam Muhammad Ibn Saud Islamic University
College of Economics and Administration Sciences
Department of Finance and Investment

جامعة الإمام محمد بن سعود الإسلامية
كلية الاقتصاد والعلوم الإدارية
قسم التمويل والاستثمار

Course **Financial Mathematics**

Unit course **FIN 118**

Number Unit **6**

Unit Subject **Applications of Matrices**

Dr. Lotfi Ben Jedidia
Dr. Imed Medhioub



we will see in this unit

- ✓ Determinant of matrices
- ✓ Some Properties of Determinants
- ✓ Some applications of determinant of matrices



LEARNING OUTCOMES

At the end of this chapter, you should be able to:

1. Identify square matrices and its regularity.
2. Find the determinant of matrices
3. Solve the system of linear equations by Cramer's Rule.



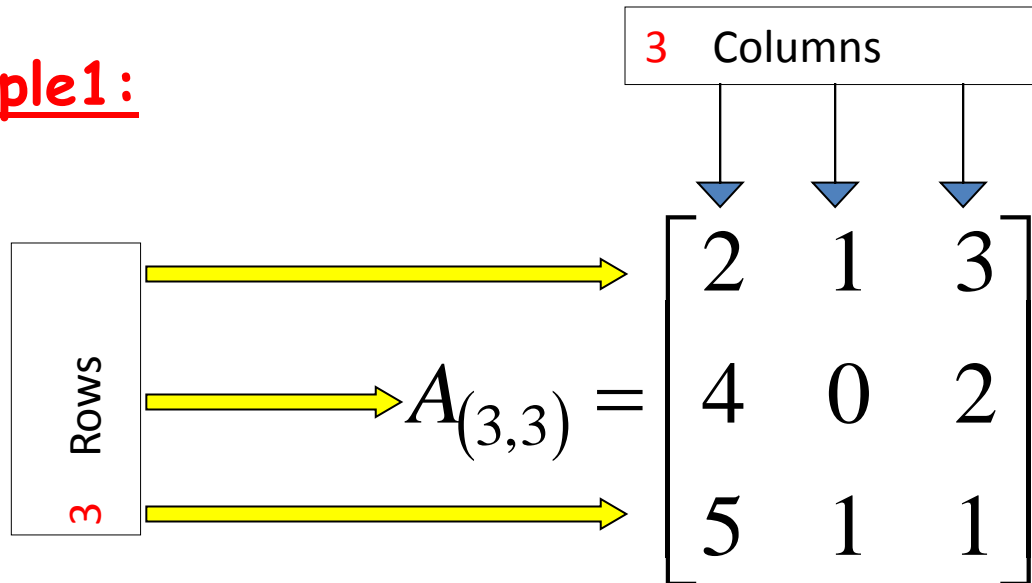
Square Matrices

Definition:

In Mathematics, square matrices play prominent role in the application of matrix algebra to real-world problems.

- A square matrix is a matrix with the same number of rows and columns. An n -by- n matrix is known as a square matrix of order n . Any two square matrices of the same order can be added and multiplied.

Example 1:



Example of Square Matrix

- Food shopping online: people go online to shop three items and have them delivered to their homes.
- Cartons of eggs, bread, bags of rice were ordered online and the people left their address for delivery.
- A selection of orders may look like this:

Order Address	Carton of eggs	Bread	Rice
Al Wuroud	2	1	3
Al Falah	4	0	2
Al Izdihar	5	1	1



Remember a Square Matrix notation

A square matrix is defined by its order which is always number of rows by number of columns.

$$A_{(n,n)} = \begin{bmatrix} a_{11} & a_{12} & a_{13}\dots & a_{1n} \\ a_{21} & a_{22} & a_{23}\dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & a_{n3}\dots & a_{nn} \end{bmatrix}$$

- A horizontal set of elements is called a row
- A vertical set is called a column
- First subscript refers to the row number
- Second subscript refers to column number



Determinant of matrices

Definition 1:

The determinant of square matrix A , denoted $\det(A)$, or $|A|$, is a number that is evaluated by all elements of A .

Determinant of order 2

The determinant of a 2×2 matrix is the difference between the product of the major diagonal elements and the product of the minor diagonal elements .

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$



More Examples

Evaluate the determinant of each matrix:

1/ $A = \begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$

2/ $B = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

3/ $C = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

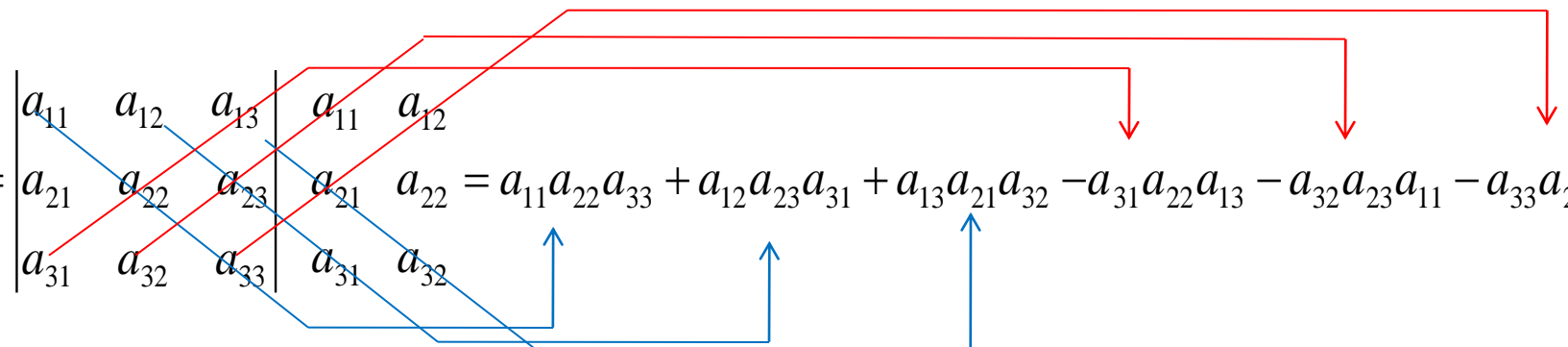
4/ $D = \begin{bmatrix} -2 & -1 \\ -2 & -5 \end{bmatrix}$



Determinant of matrices

Determinant of order 3:

- To find the determinant of a 3×3 matrix, first recopy the first two columns. Then we obtain 3 **major** diagonal elements and 3 **minor** diagonal elements (**Rule of Sarrus**).
- The determinant of a 3×3 matrix is the difference between the sum of the products of the **major** diagonal elements and the sum of the products of the **minor** diagonal elements.



$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$$

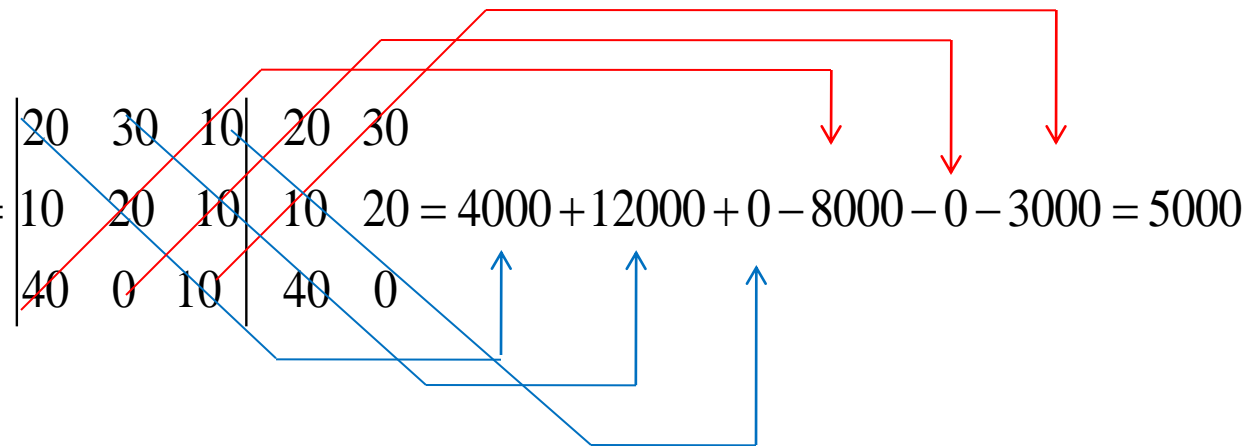


Determinant of matrices

Example:

Find the determinant of the following matrix

$$M = \begin{bmatrix} 20 & 30 & 10 \\ 10 & 20 & 10 \\ 40 & 0 & 10 \end{bmatrix}$$

$$\det(M) = \begin{vmatrix} 20 & 30 & 10 \\ 10 & 20 & 10 \\ 40 & 0 & 10 \end{vmatrix} = \begin{vmatrix} 20 & 30 \\ 10 & 20 \end{vmatrix} \begin{vmatrix} 20 & 30 \\ 40 & 0 \end{vmatrix} - \begin{vmatrix} 20 & 10 \\ 10 & 10 \end{vmatrix} \begin{vmatrix} 20 & 30 \\ 40 & 0 \end{vmatrix} + \begin{vmatrix} 20 & 10 \\ 40 & 0 \end{vmatrix} \begin{vmatrix} 10 & 20 \end{vmatrix} = 4000 + 12000 + 0 - 8000 - 0 - 3000 = 5000$$




Determinant of matrices

Examples:

Evaluate the determinant of these matrices:

$$P_1 = \begin{vmatrix} 1460 & 30 & 10 \\ 990 & 20 & 10 \\ 1300 & 0 & 10 \end{vmatrix}$$

$$P_2 = \begin{vmatrix} 20 & 1460 & 10 \\ 10 & 990 & 10 \\ 40 & 1300 & 10 \end{vmatrix}$$

$$P_3 = \begin{vmatrix} 20 & 30 & 1460 \\ 10 & 20 & 990 \\ 40 & 0 & 1300 \end{vmatrix}$$



Some Properties of Determinants

P1: $\det(A) = \det(A^T)$

P2: If all entries of any row or column is zero, then $\det(A) = 0$

P3: If two rows or two columns are identical, or linearly dependent then $\det(A) = 0$

P4: If A is a diagonal matrix or upper triangular or lower triangular matrix, then $\det(A)$ is equal to the product of all diagonal elements.

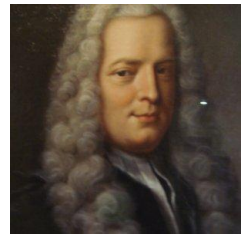
$$|A| = \prod_{i=1}^n a_{ii}$$

P5: if $|A| \neq 0$, then A is regular and invertible



Some applications of determinant of matrices

- Mr. Cramer tells us that we can use determinants to solve a linear system. (No elimination; No substitution!)



Gabriel
Cramer
(1750ish)

Cramer's Rule on a System of Two Equations

Let A be the coefficient matrix for the system:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases} \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

If $\det(A) \neq 0$, then the system has one solution, and

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{|A|}$$

$$x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{|A|}$$

$$S = \{(x_1, x_2)\}$$



Cramer's Rule on a System of Two Equations



Example

Solve the system using Cramer's Rule.

$$\begin{cases} 2x_1 + 3x_2 = 5 \\ 3x_1 + 5x_2 = 12 \end{cases}$$

Step1:

The coefficient matrix for the system and its determinant are:

$$A = \begin{bmatrix} & \\ & \end{bmatrix}, \det(A) = \begin{vmatrix} & \\ & \end{vmatrix} =$$

Step2:

$$x_1 = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{\begin{vmatrix} & \\ & \end{vmatrix}} =$$

$$x_2 = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{\begin{vmatrix} & \\ & \end{vmatrix}} =$$

$$S = \{(-11, 9)\}$$



Cramer's Rule on a System of Three Equations

Let A be the coefficient matrix for the system:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

If $\det(A) \neq 0$, then the system has one solution, and

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{|A|}$$

$$x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{|A|}$$

$$x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{|A|}$$

$$S = \{(x_1, x_2, x_3)\}$$



Cramer's Rule on a System of Three Equations



Example:

Solve the system using Cramer's Rule.

$$\begin{cases} 2x_1 + 3x_2 + x_3 = 146 \\ x_1 + 2x_2 + x_3 = 99 \\ 4x_1 + x_3 = 130 \end{cases}$$

Step1:

The coefficient matrix for the system and its determinant are:

$$A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$A = \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} =$$



Cramer's Rule on a System of Three Equations



Step2:

$$x_1 = \frac{\begin{vmatrix} \phantom{a_{11}} & \phantom{a_{12}} & \phantom{a_{13}} \\ \phantom{a_{21}} & \phantom{a_{22}} & \phantom{a_{23}} \\ \phantom{a_{31}} & \phantom{a_{32}} & \phantom{a_{33}} \end{vmatrix}}{}$$

$$x_2 = \frac{\begin{vmatrix} \phantom{a_{11}} & \phantom{a_{12}} & \phantom{a_{13}} \\ \phantom{a_{21}} & \phantom{a_{22}} & \phantom{a_{23}} \\ \phantom{a_{31}} & \phantom{a_{32}} & \phantom{a_{33}} \end{vmatrix}}{}$$

$$x_3 = \frac{\begin{vmatrix} \phantom{a_{11}} & \phantom{a_{12}} & \phantom{a_{13}} \\ \phantom{a_{21}} & \phantom{a_{22}} & \phantom{a_{23}} \\ \phantom{a_{31}} & \phantom{a_{32}} & \phantom{a_{33}} \end{vmatrix}}{}$$

$$S = \{(25, 22, 30)\}$$



Time to Review !

- ✓ Matrices are used to transcript information in a system of equations
- ✓ The determinants of Matrices can be used to solve a linear system. (No elimination; No substitution ! **Cramer rule**)



we will see in the next unit

- ✓ The "arithmetic sequences" and "arithmetic series".
- ✓ The "Geometric sequences" and "Geometric series".
- ✓ Solve some questions for real world situations in order to solve problems, especially economic and financial.

