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Course	**Financial Mathematics**
Unit course	**FIN 118**
Number Unit	**6**
Unit Subject	**Applications of Matrices**

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we will see in this unit

- ✓ Determinant of matrices
- ✓ Some Properties of Determinants
- ✓ Some applications of determinant of matrices

LEARNING OUTCOMES

At the end of this chapter, you should be able to:

1. Identify square matrices and its regularity.
2. Find the determinant of matrices
3. Solve the system of linear equations by Cramer's Rule.

Square Matrices

Definition:

In Mathematics, square matrices play prominent role in the application of matrix algebra to real-world problems.

- A square matrix is a matrix with the same number of rows and columns. An n-by-n matrix is known as a square matrix of order n. Any two square matrices of the same order can be added and multiplied.

Example1:

3 Columns

3 Rows

$$A_{(3,3)} = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 0 & 2 \\ 5 & 1 & 1 \end{bmatrix}$$

Example of Square Matrix

- Food shopping online: people go online to shop three items and have them delivered to their homes.
- Cartons of eggs, bread, bags of rice were ordered online and the people left their address for delivery.
- A selection of orders may look like this:

Order / Address	Carton of eggs	Bread	Rice
Al Wuroud	2	1	3
Al Falah	4	0	2
Al Izdihar	5	1	1

Remember a Square Matrix notation

A square matrix is defined by its order which is always number of rows by number of columns.

$$A_{(n,n)} = \begin{bmatrix} a_{11} & a_{12} & a_{13}\cdots & a_{1n} \\ a_{21} & a_{22} & a_{23}\cdots & a_{2n} \\ . & . & . & . \\ a_{n1} & a_{n2} & a_{n3}\cdots & a_{nn} \end{bmatrix}$$

- A horizontal set of elements is called a row
- A vertical set is called a column
- First subscript refers to the row number
- Second subscript refers to column number

Determinant of matrices

Definition 1:

The determinant of square matrix A, denoted det(A), or $|A|$, is a number that is evaluated by all elements of A.

Determinant of order 2

The determinant of a 2x2 matrix is the difference between the product of the major diagonal elements and the product of the minor diagonal elements .

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

More Examples

Evaluate the determinant of each matrix:

1/ $A = \begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$

2/ $B = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

3/ $C = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

4/ $D = \begin{bmatrix} -2 & -1 \\ -2 & -5 \end{bmatrix}$

Determinant of matrices

Determinant of order 3:

- To find the determinant of a 3 x 3 matrix, first recopy the first two columns. Then we obtain 3 major diagonal elements and 3 minor diagonal elements (**Rule of Sarrus**).
- The determinant of a 3x3 matrix is the difference between the sum of the products of the major diagonal elements and the sum of the products of the minor diagonal elements .

$$\det(A)=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$$

Determinant of matrices

Example:

Find the determinant of the following matrix

$$M = \begin{bmatrix} 20 & 30 & 10 \\ 10 & 20 & 10 \\ 40 & 0 & 10 \end{bmatrix}$$

$$\det(M) = \begin{vmatrix} 20 & 30 & 10 \\ 10 & 20 & 10 \\ 40 & 0 & 10 \end{vmatrix} \begin{matrix} 20 & 30 \\ 10 & 20 \\ 40 & 0 \end{matrix} = 4000 + 12000 + 0 - 8000 - 0 - 3000 = 5000$$

Determinant of matrices

Examples:

Evaluate the determinant of theses matrices:

$$P_1 = \begin{vmatrix} 1460 & 30 & 10 \\ 990 & 20 & 10 \\ 1300 & 0 & 10 \end{vmatrix}$$

$$P_2 = \begin{vmatrix} 20 & 1460 & 10 \\ 10 & 990 & 10 \\ 40 & 1300 & 10 \end{vmatrix}$$

$$P_3 = \begin{vmatrix} 20 & 30 & 1460 \\ 10 & 20 & 990 \\ 40 & 0 & 1300 \end{vmatrix}$$

Some Properties of Determinants

P1: det (A) = det (A^T)

P2: If all entries of any row or column is zero, then det (A) = 0

P3: If two rows or two columns are identical, or linearly dependent then det (A) = 0

P4: If A is a diagonal matrix or upper triangular or lower triangular matrix, then det(A) is equal to the product of all diagonal elements.

$$|A| = \underset{i=1}{\overset{n}{\pi}} a_{ii}$$

P5: if $|A| \neq 0$, then A is regular and invertible

Some applications of determinant of matrices

Gabriel Cramer (1750ish)

- Mr. Cramer tells us that we can use determinants to solve a linear system. (No elimination; No substitution!)

Cramer's Rule on a System of Two Equations

Let A be the coefficient matrix for the system:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases} \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

If $\det(A) \neq 0$, then the system has one solution, and

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{|A|} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{|A|}$$

$S = \{(x_1, x_2)\}$

Cramer's Rule on a System of Two Equations

Example

Solve the system using Cramer's Rule.

$$\begin{cases} 2x_1 + 3x_2 = 5 \\ 3x_1 + 5x_2 = 12 \end{cases}$$

Step1:

The coefficient matrix for the system and its determinant are:

$$A = \begin{bmatrix} \quad & \quad \end{bmatrix} , \quad \det(A) = \begin{vmatrix} \quad & \quad \end{vmatrix} =$$

Step2:

$$x_1 = \frac{\begin{vmatrix} \quad \end{vmatrix}}{\quad} = \qquad x_2 = \frac{\begin{vmatrix} \quad \end{vmatrix}}{\quad} =$$

S = {(-11, 9)}

Cramer's Rule on a System of Three Equations

Let A be the coefficient matrix for the system:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases} \qquad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

If det(A) $\neq$ 0, then the system has one solution, and

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{|A|} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{|A|} \qquad x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{|A|}$$

S = {(x_1 , x_2, x_3)}

Cramer's Rule on a System of Three Equations

Example:

Solve the system using Cramer's Rule.

$$\begin{cases} 2x_1 + 3x_2 + x_3 = 146 \\ x_1 + 2x_2 + x_3 = 99 \\ 4x_1 + x_3 \qquad = 130 \end{cases}$$

Step1:

The coefficient matrix for the system and its determinant are:

$$A = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} \qquad A = \begin{vmatrix} \quad & \quad & \quad \end{vmatrix} =$$

Cramer's Rule on a System of Three Equations

Step2:

$$x_1 = \frac{\left| \quad \right|}{\quad} =$$

$$x_2 = \frac{\left| \quad \right|}{\quad} =$$

$$x_3 = \frac{\left| \quad \right|}{\quad} =$$

S = {(25, 22, 30)}

Time to Review !

- ✓Matrices are used to transcript information in a system of equations
- ✓ The determinants of Matrices can be used to solve a linear system. (No elimination; No substitution ! Cramer rule)

we will see in the next unit

- ✓ The "arithmetic sequences" and "arithmetic series".
- ✓ The "Geometric sequences" and "Geometric series".
- ✓ Solve some questions for real world situations in order to solve problems, especially economic and financial.