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قسم التمويل والاستثمار

Course	**Financial Mathematics**
Unit course	**FIN 118**
Number Unit	**4**
Unit Subject	**Integral Calculus**

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We will see in this unit

1. Integral calculus : Definition
2. Indefinite integral
3. Definite integral
4. Some rules of integral
5. Area between two curves

Learning Outcomes

At the end of this chapter, you should be able to:

1.Understand what is meant by “integral of function”.

2. Find definite or indefinite integrals.

3. Calculate the Area Between Two Curves.

Integral calculus

Frequently, we know the rate of change of a function $\left(f^{'}(x)\right)$ and wish to find the original function $f(x)$. Reversing the process of differentiation and finding the original function from the derivative is called integration or anti-differentiation. The original function, $f(x)$, is called the integral or antiderivative of $f^{'}(x.)$

Thus, we have $\int f^{'}(x)dx = f(x) + c$

Integral calculus

Example 1:

1/ Find the derivative of $f_1(x)=c$, $f_2(x)=x$, $f_3(x)=x^2$

2/ Find the antiderivative of the results of question 1.

Solution:

1/ $f_1'(x)=0$, $f_2'(x)=1$, $f_3'(x)=2x$

2/ $\int f_1'(x)dx = \int 0dx = c$, $\int f_2'(x)dx = \int 1dx = x + c$

$\int f_3'(x)dx = \int 2xdx = x^2 + c$

Indefinite Integral

- The indefinite integral of a function is a function defined as : $\int f(x)dx = F(x) + c$
- Every antiderivative F of f must be of the form $F(x) = G(x) + c$, where c is a constant (constant of integration)

!!!

$$\int 2x\,dx = \underbrace{x^2 + c}$$

Represents every possible antiderivative of 2x.

Definite integral

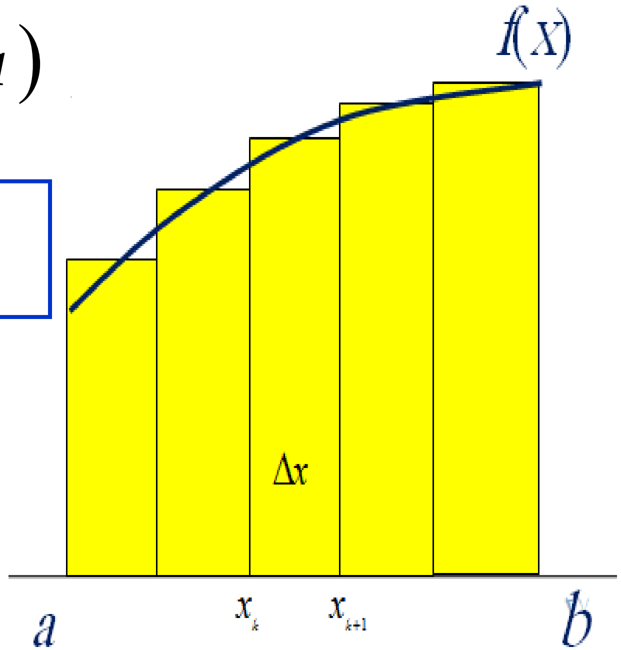
If f is a continuous function, the definite integral of f from a to b is defined as:

$$\int_a^b f(x)dx = F(b) - F(a)$$

An integral = Area under a curve

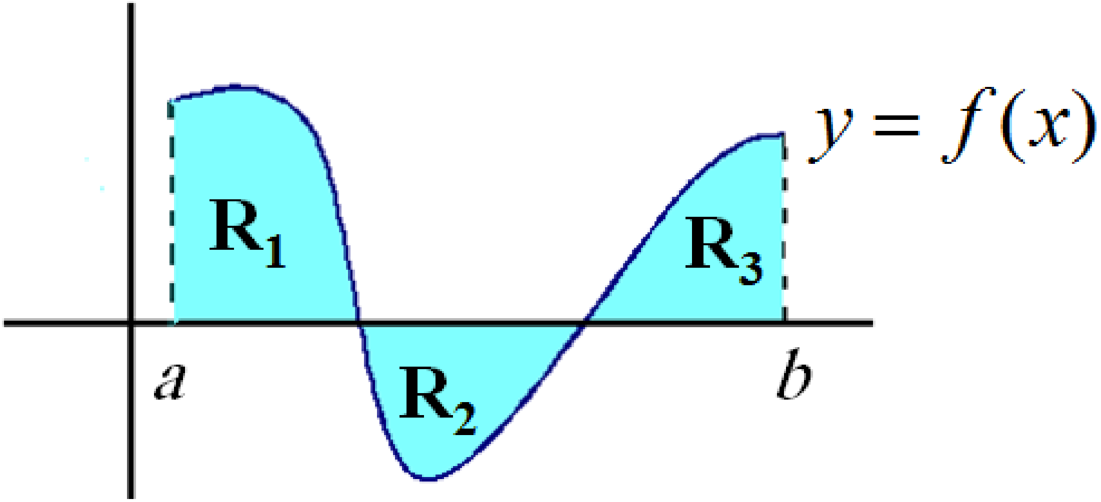
$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

$$\Delta x = \frac{b-a}{n} = x_{k+1} - x_k$$



Integral Calculus

Exemple1 :



$$\int_a^b f(x)dx = \text{Area of } R_1 - \text{Area of } R_2 + \text{Area of } R_3$$

Integral Calculus

Example2:

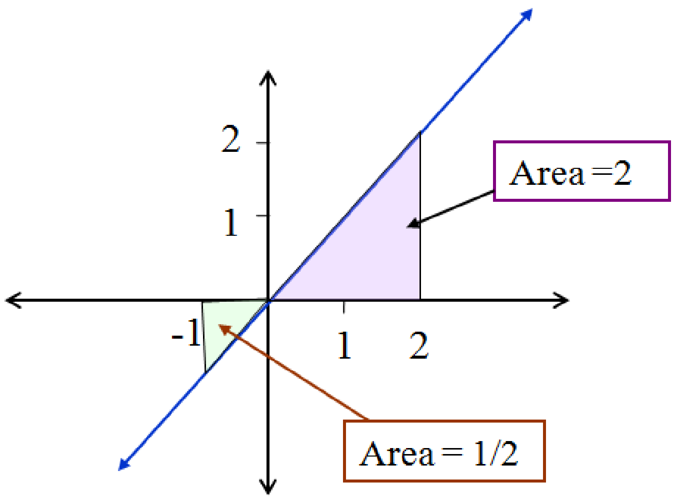
1/ Calculate algebraically the integral

$$\int_{-1}^{2} x\,dx = \left[\frac{x^2}{2}\right]_{-1}^{2} = \frac{(2)^2}{2} - \frac{(-1)^2}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

2/ Use geometry to

compute the same integral

$$\int_{-1}^{2} x\,dx = 2 - \frac{1}{2} = \frac{3}{2}$$



Some rules of integration

To simplify the determination of antiderivatives we can use the following rules.

1/ $\int dx = x + c$

2/ $\int k dx = kx + c$

3/ $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

4/ $\int \frac{1}{x} dx = \ln|x| + c$

5/ $\int b^x dx = \frac{b^x}{\ln(b)} + c$

6/ $\int e^x dx = e^x + c$

Some rules of integration

7/ $$\int (f \pm g)\,dx = \int f\,dx \pm \int g\,dx$$

8/ $$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C \quad (n \neq -1)$$

9/ $$\int (ax+b)^{-1}\,dx = \frac{1}{a}\ln|ax+b| + C$$

10/ $$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$$

11/ $$\int c^{ax+b}\,dx = \frac{1}{a\ln c}c^{ax+b} + C$$

More examples

1/ $$\int 2x^3dx = 2\int x^3dx = 2\frac{x^4}{4} + C = \frac{x^4}{2} + C$$

2/ $$\int (6y^5 + 3y)dy = y^6 + \frac{3}{2}y^2 + c$$

3/ $$\int \left(\frac{1}{x} - e^{2x}\right)dx = \ln|x| - \frac{1}{2}e^{2x} + c$$

4/ $$\int (6x-1)^2 dx = \int (36x^2 - 12x + 1)dx$$

$$= \frac{36}{3}x^3 - \frac{12}{2}x^2 + x + c$$

$$= 12x^3 - 6x^2 + x + c$$

Examples

1/ $\int_{-1}^{1}\left(x^2-7x+12\right)dx=$

2/ $\int_{0}^{-2}\left(3x^2-3\right)dx=$

3/ $\int_{0}^{3}\left(e^x\right)dx=$

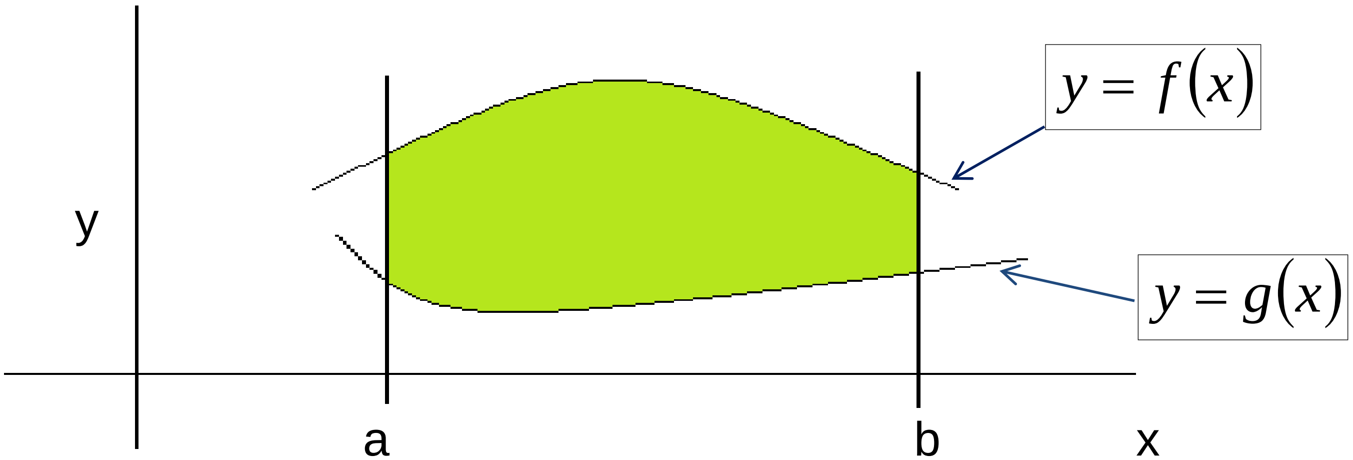
4/ $\int_{0}^{1}\left(e^{2x+3}\right)dx=$

5/ $\int_{1}^{5}\left(2x-\frac{1}{x}+1\right)dx=$

Area Between Two Curves

Let f and g be continuous functions, the area bounded above by f (x) and below by g(x) on [a, b] is:

$$R = \int_a^b \left[f(x) - g(x) \right] dx$$



Area Between Two Curves

Example:

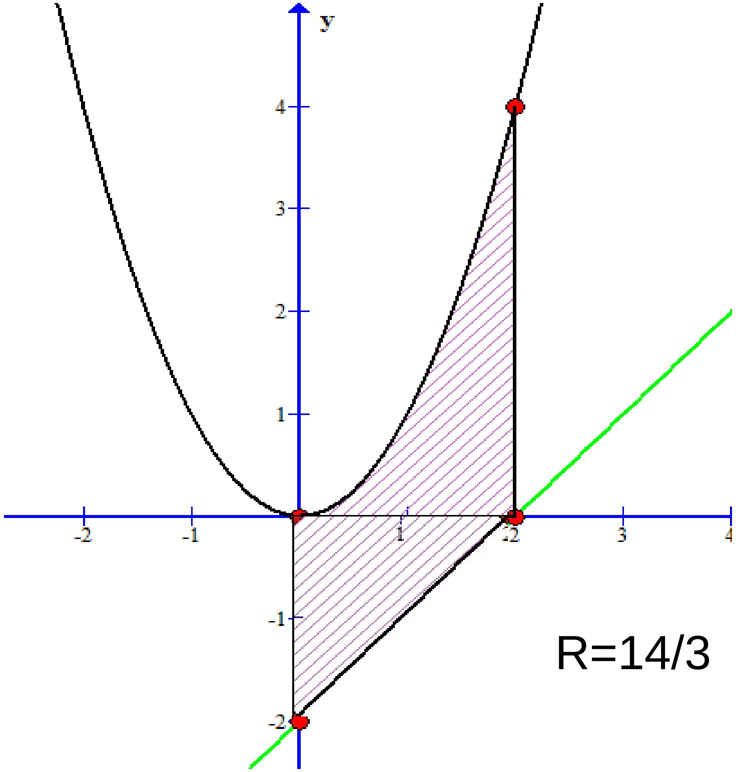
Find the area bounded by the curves

$$R = \int_0^2 [f(x) - g(x)]dx \quad \text{where}$$

$$f(x) = x^2 \quad \text{and}$$

$$g(x) = x - 2$$

R =



Time to Review !

1. By reversing the process of differentiation, we find the original function from the derivative. We call this operation integration or anti-differentiation.
2. The indefinite integral of a function is a function defined as : $\int f(x)dx = F(x) + c$
3. If f is a continuous function, the definite integral of f from a to b is defined as:

$$\int_a^b f(x)dx = F(b) - F(a)$$

we will see in the next unit

- ✓ Matrix / Matrices
- ✓ Different types of matrices
- ✓ Usual operations on matrices