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جامعة الإمام محمد بن سعود الإسلامية
كلية الاقتصاد والعلوم الإدارية
قسم التمويل والاستثمار

Course	**Financial Mathematics**
Unit course	**FIN 118**
Number Unit	**13**
Unit Subject	**Equal short term payments** **Settlement of short-term debt**

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we will see in this unit

- ✓ Meant of Equal short term payments and settlement of short-term debt
- ✓How to Calculate the amount of total payments.
- ✓How to Calculate the amount of a new settlement

Learning Outcomes

At the end of this chapter, you should be able to:

1. Apply the rule of an ordinary annuity and an annuity due to Equal short term payments and settlement of short-term debt

2. Calculate the amount of total payments and the amount of total settlement of short-term debt.

Introduction

In unit 10, we have seen that a multiple stream of cash flow that is made in an equal size and at a regular interval is known as simple annuity. Therefore we have seen that exists four types of Simple annuity: Ordinary Annuity, Annuity Due (unit10), Deferred Annuity, and Perpetuity. So we have applied the compound interest to this type of annuity to calculate future or present value of the amount of all stream.

In this unit we attempt to apply the simple interest rather than compound interest for multiple stream of cash flow known as Equal short term payments. We also explain how to apply the settlement of short-term debt via some practical examples.

1/ Equal short term payments

Definition: Equal short term payments is a series of equal cash payments made at the end or at beginning of each simple period.

Examples :

i/ When you buy some house wares in monthly installment

ii/ When financing operational activities of Firm.

Ordinary Payments / Payments due

Definition: Ordinary payments, called the reimbursement installments, which payments are to be paid at the end of each period of time, it has paid at the end of every month or every two months, or every 3 months or etc.

Definition: Payments Due, called the reimbursement installments, which payments are to be paid at the beginning of each period of time, it has paid at the beginning of every month or every two months, or every 3 months or etc.

How to calculate Total of installments?

Generally, the total of installments is equal to the principal plus interest. It is determined as follow:

$$\boxed{Total\ Installments = Principal + Interest}$$

$$\boxed{Total\ Installments = PMT \times n + PMT \times i \times T}$$

$$\boxed{Total\ Installments = PMT\left(n + i \times T\right)}$$

* PMT: the <u>installment</u> to be paid at each period,
* i : interest rate per period,
* n : number of payments,
* T: the number of time periods.

How to calculate Total of installments?

The number of time periods (T) is generally, function of the number of payments.

$$T = \frac{n}{2}\begin{pmatrix} \textit{Number of periods of the first payment until maturity} \\ + \\ \textit{Number of periods of the last payment until maturity} \end{pmatrix}$$

Example1: Suppose you plan to deposit \$1000 at the end of each month into an account for one year. If the account pays a simple interest equal to 15% annually, what is the value of your deposits at the end of the year?

Solution:

PMT = 1000, n =12, i = 0.15, $T = \frac{12}{2}(11 + 0) = 66$

$$Total\ Installments = 1000\left(12 + \frac{0.15}{12} \times 66\right) = 12825$$

How to calculate Total of installments?

Example2: Suppose you plan to deposit $1000 at the beginning of each month into an account for one year. If the account pays a simple interest equal to 15% annually, what is the value of your deposits at the end of the year?

Solution:

PMT = 1000, n =12, i = 0.15, $\boxed{T = \frac{12}{2}(12+1) = 78}$

$$\boxed{Total\ Installments = 1000\left(12 + \frac{0.15}{12} \times 78\right) = 12975}$$

How to calculate Total of installments?

Example3: Suppose you plan to deposit $1000 in the middle of each month into an account for one year. If the account pays a simple interest equal to 15% annually, what is the value of your deposits at the end of the year?

Solution:

PMT = 1000, n =12, i = 0.15, $\boxed{T = \frac{12}{2}\left(11.5 + 0.5\right) = 72}$

$$\boxed{Total\ Installments = 1000\left(12 + \frac{0.15}{12} \times 72\right) = 12900}$$

How to calculate Total of installments?

Example4: Suppose you plan to deposit \$1000 at the end of each month into an account for six months. If the account pays a simple interest equal to 15% annually, what is the value of your deposits at the end of the year?

Solution:

PMT = 1000, n =6, i = 0.15,

$$\boxed{T = \frac{6}{2}(11+6) = 51}$$

$$\boxed{Total\ Installments = 1000\left(6 + \frac{0.15}{12} \times 51\right) = 6637.5}$$

How to calculate Total of installments?

Example5: Suppose you plan to deposit $1000 at the end of each two months into an account for one year and half. If the account pays a simple interest equal to 15% annually, what is the value of your deposits at the end of the period?

Solution:

PMT = 1000, n =18/2 = 9, i = 0.15,

$$\boxed{T = \frac{9}{2}(16+0) = 72}$$

$$\boxed{Total\ Installments = 1000\left(9 + \frac{0.15}{12} \times 72\right) = 9900}$$

2/ the settlement of short-term debt

Definition: The settlement of short-term debt is intended to an agreement between the debtor and the creditor to replace old debt by new debt. Therefore the agreement includes a method of replacement and general rule used is the equality between the value of old debt and the value of new debt at a specific date which is called the settlement date.

Rule :

The value of the old debt at the date of settlement

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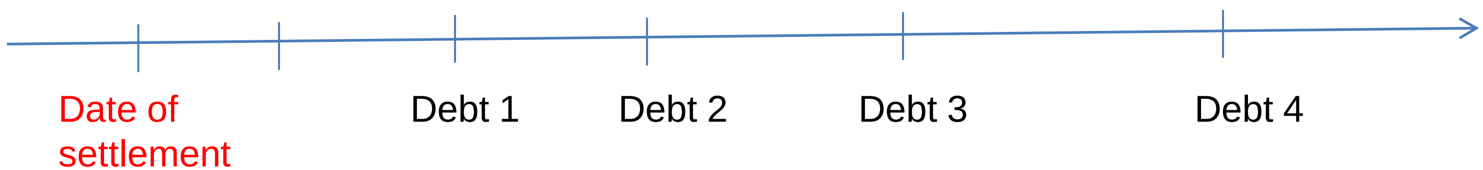
The value of the new debt at the date of settlement

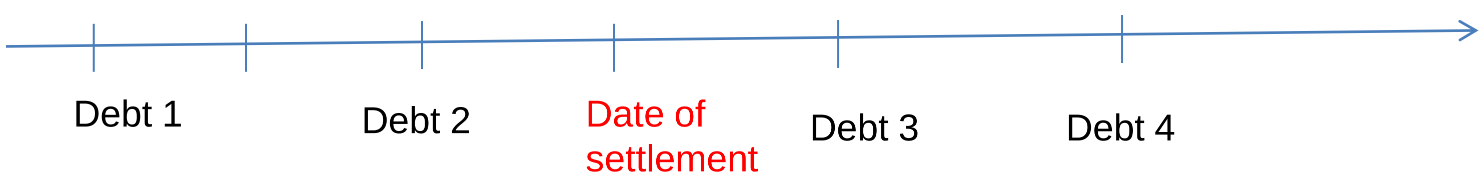
How to calculate the value of New Debt ?

Generally, we have three cases:

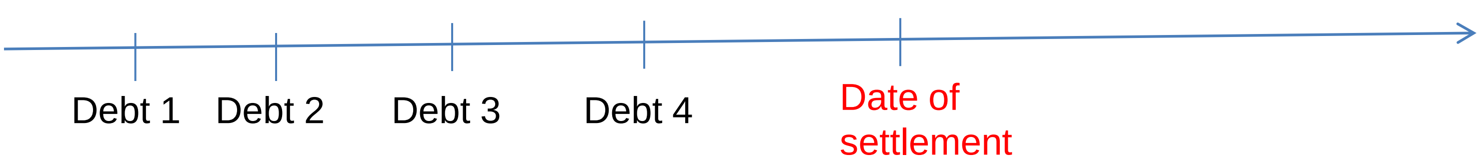
Case 1 : the settlement date is before all dates of maturity. So we must calculate the present value of each batch at the date of settlement.



Case 2 : the settlement date is before one date of maturity. So we must calculate the PV of each batch after the date of settlement and FV of each batch before the date of settlement .



Case 3: the settlement date is after all dates of maturity, we must calculate the future value of each batch at the date of settlement



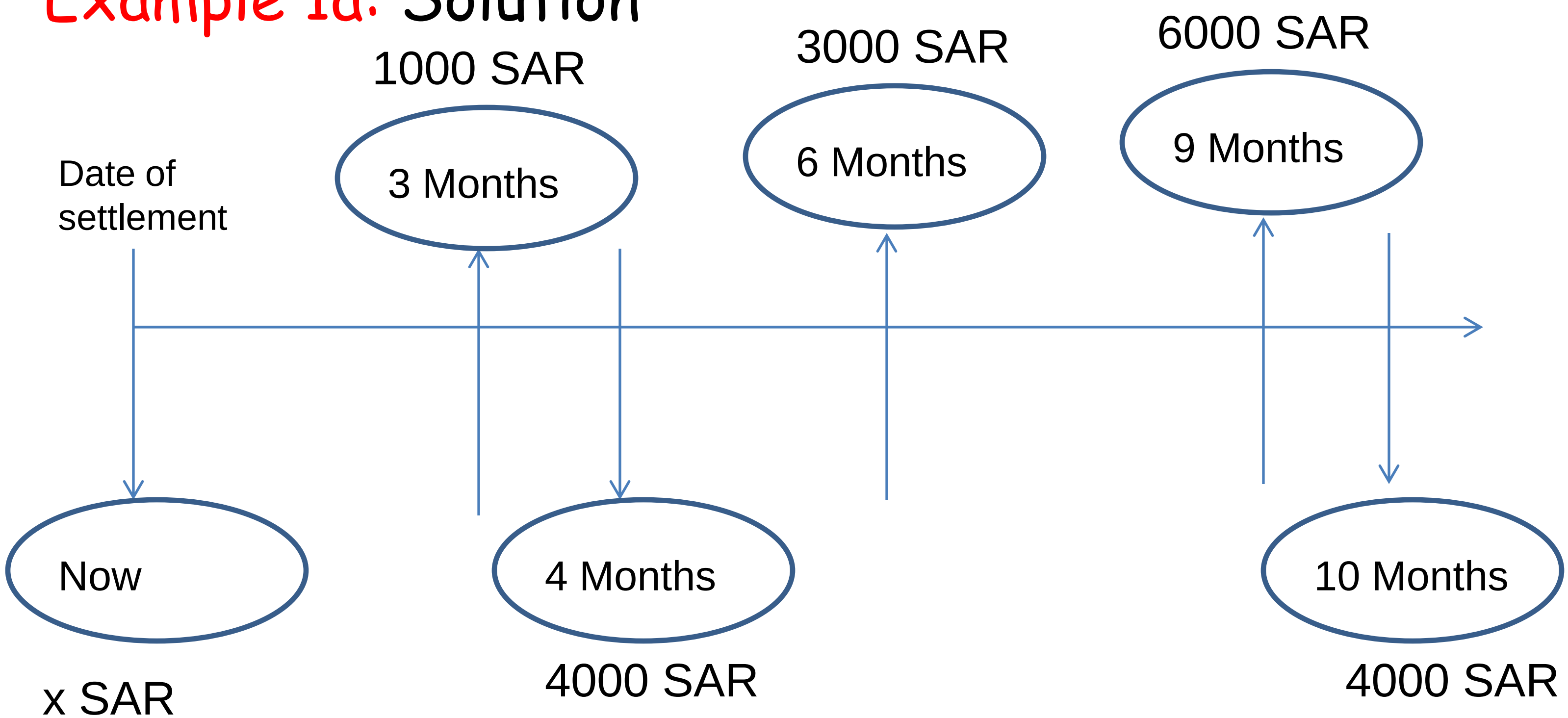
How to calculate the value of New Debt and installments ?

Example 1a: Someone owes the following amounts:
1000 SAR payable after 3 months
3000 SAR payable after 6 months
6000 SAR payable after 9 months
It was agreed with the creditor to sign two new promissory notes with the same amount fixed at 4000 SAR. The first worth after 4 months, the second after 10 months and pay the rest owed cash immediately.

Calculate the amount of cash paid by the debtor if we set the discount rate at 5% per annum.

How to calculate the value of New Debt and installments ?

Example 1a: Solution



$$PV_1 = 1000 \times \left[1 - 0.05 \times \frac{3}{12}\right] = 987.5$$

$$PV_2 = 3000 \times \left[1 - 0.05 \times \frac{6}{12}\right] = 2925$$

$$PV_3 = 6000 \times \left[1 - 0.05 \times \frac{9}{12}\right] = 5775$$

$$PV(olddebt) = 9687.5$$

How to calculate the value of New Debt and installments ?

Example 1a: Solution (continued)

$$PV_1 = x$$

$$PV_2 = 4000\left[1 - 0.05 \times \frac{4}{12}\right] = 3933.33$$

$$PV_3 = 4000\left[1 - 0.05 \times \frac{10}{12}\right] = 3833.33$$

$$PV(Newdebt) = x + 3933.33 + 3833.33$$

$$PV(Newdebt) = 7766.66 + x$$

$$PV(olddebt) = PV(Newdebt)$$

$$7766.66 + x = 9687.5 \Rightarrow x = 9687.5 - 7766.66 = 1920.84$$

The amount of cash paid by the debtor = 1920.84 SAR

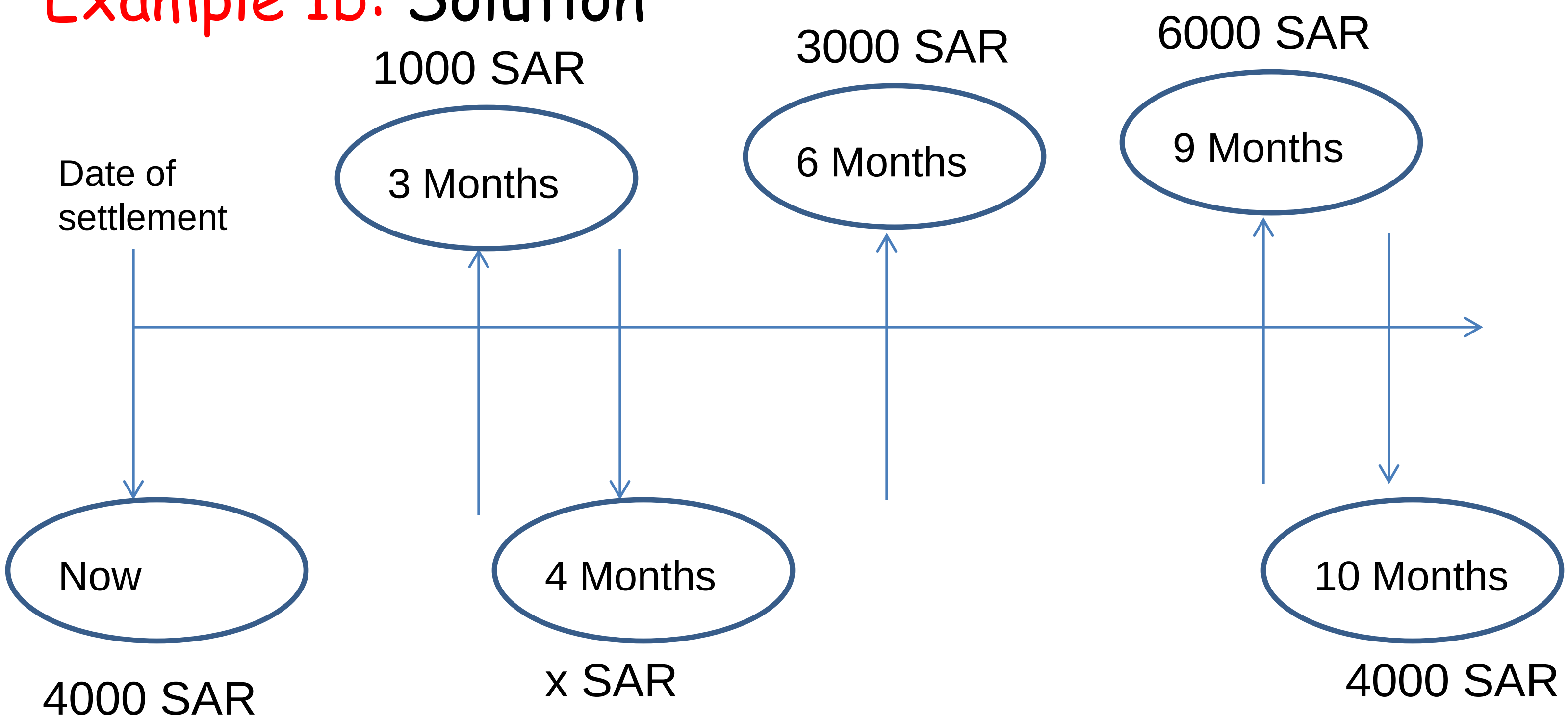
How to calculate the value of New Debt and installments ?

Example 1b: Someone owes the following amounts:
1000 SAR payable after 3 months
3000 SAR payable after 6 months
6000 SAR payable after 9 months

It was agreed with the creditor to pay immediately 4000 SAR and sign two new promissory notes. If we set the discount rate at 5% per annum and the value of the second promissory note equal to 4000 SAR, calculate the amount of the first promissory note.

How to calculate the value of New Debt and installments ?

Example 1b: Solution



$$PV_1 = 1000 \times \left[1 - 0.05 \times \frac{3}{12} \right] = 987.5$$

$$PV_2 = 3000 \times \left[1 - 0.05 \times \frac{6}{12} \right] = 2925$$

$$PV_3 = 6000 \times \left[1 - 0.05 \times \frac{9}{12} \right] = 5775$$

$$PV(olddebt) = 9687.5$$

How to calculate the value of New Debt and installments ?

Example 1b: Solution (continued)

$$PV_1 = 4000$$

$$PV_2 = x\left[1 - 0.05 \times \frac{4}{12}\right] = 0.983x$$

$$PV_3 = 4000\left[1 - 0.05 \times \frac{10}{12}\right]$$
$$= 3833.33$$

$$PV(Newdebt) = 4000 + 0.983x + 3833.33$$

$$PV(Newdebt) = 7833.33 + 0.983x$$

$$PV(olddebt) = PV(Newdebt)$$

$$7833.33 + 0.983x = 9687.5 \Rightarrow x = \frac{9687.5 - 7833.33}{0.983} = 1886.24$$

The value of the first promissory note = 1886.24 SAR

How to calculate the value of New Debt and installments ?

Example 1c: Someone owes the following amounts:

1000 SAR payable after 3 months

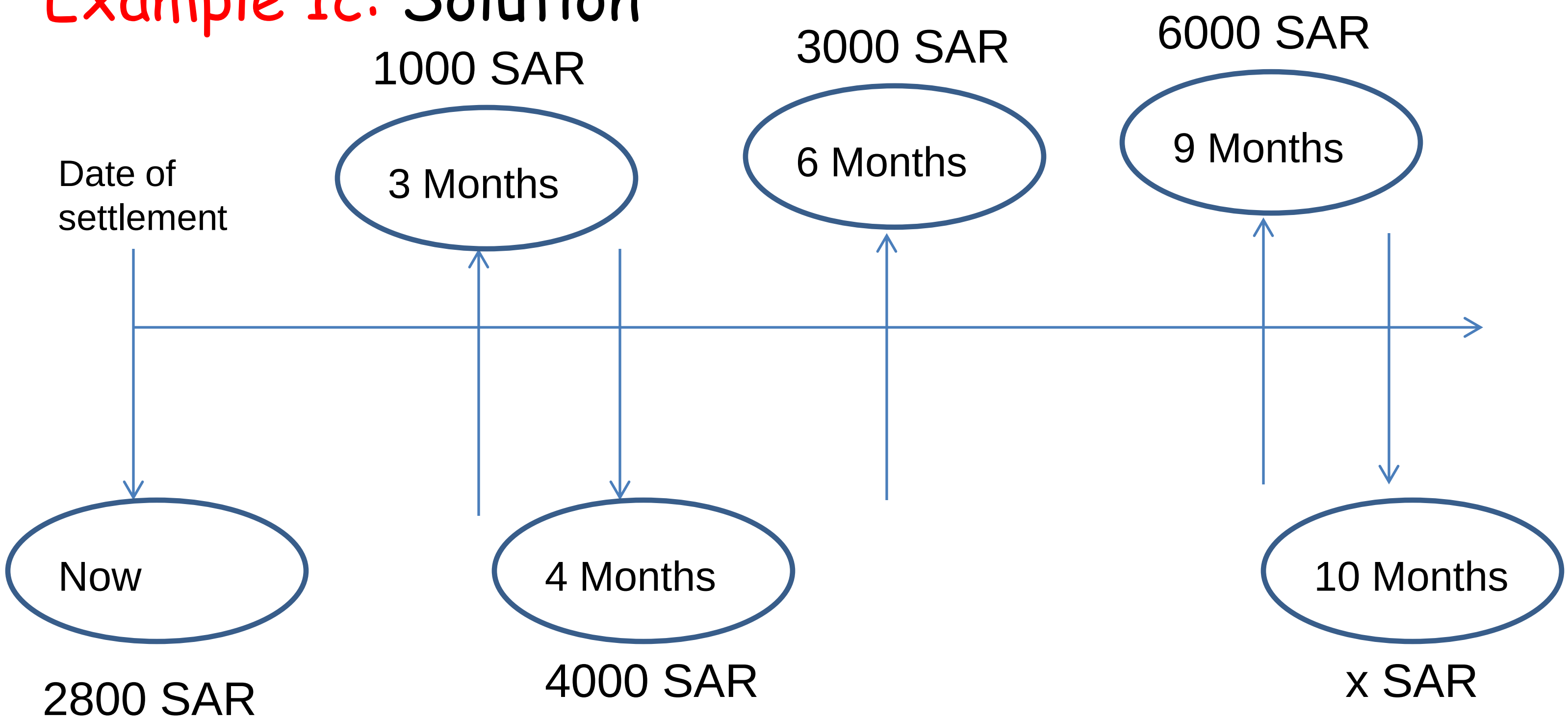
3000 SAR payable after 6 months

6000 SAR payable after 9 months

It was agreed with the creditor to pay immediately the amount of 2800 SAR, and the rest of new debt is divided on two promissory notes. The value of the first promissory note is equal to 4000 SAR. If the first is payable after 4 months, and the second after 10 months, calculate the nominal value of the second new promissory note if the discount rate was fixed at 5% annually.

How to calculate the value of New Debt and installments ?

Example 1c: Solution



$$PV_1 = 1000 \times \left[1 - 0.05 \times \frac{3}{12}\right] = 987.5$$

$$PV_2 = 3000 \times \left[1 - 0.05 \times \frac{6}{12}\right] = 2925$$

$$PV_3 = 6000 \times \left[1 - 0.05 \times \frac{9}{12}\right] = 5775$$

$$PV(olddebt) = 9687.5$$

How to calculate the value of New Debt and installments ?

Example 1c: Solution (continued)

$$PV_1 = 2800$$

$$PV_2 = 4000\left[1 - 0.05 \times \frac{4}{12}\right] = 3933.33$$

$$PV_3 = x\left[1 - 0.05 \times \frac{10}{12}\right]$$
$$= 0.958x$$

$$PV(Newdebt) = 2800 + 3933.33 + 0.958x$$

$$PV(Newdebt) = 6733.33 + 0.958x$$

$$PV(olddebt) = PV(Newdebt)$$

$$6733.33 + 0.958x = 9687.5 \Rightarrow x = \frac{9687.5 - 6733.33}{0.958} = 3083.69$$

The value of the second promissory note = 3083.69 SAR

Time to Review!

Equal short term payments/ Settlement of short-term debt

Ordinary Payments / Payment Due

$$\boxed{Total\ Installments = Principal + Interest}$$

$$\boxed{Total\ Installments = PMT\left(n + i \times T\right)}$$

$$\boxed{T = \frac{n}{2}\left(Duration\ of\ the\ first\ payment + Duration\ of\ the\ last\ payment\right)}$$

the settlement of short-term debt

The value of the old debt at the date of settlement

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The value of the new debt at the date of settlement

الخاتمة

تم بحمد الله إتمام البرنامج

وإن تبقى من الوقت شيء فسوف نقدم تطبيقات عملية لحساب فوائد المرابحة، وفوائد البيع بالتقسيط، وفوائد الإيجار المنتهي بالتمليك

نشكركم على حسن المتابعة ونتمنى لكم النجاح والتوفيق